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What if beta is not stable? Applying the Kalman filter to risk estimates of top US companies over the long time horizon

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Abstract

The main objective of this paper is to examine the Kalman approach to estimate the time-varying CAPM beta on the US stock market over the long time horizon of thirty-one years. We investigate the beta estimates on the basis of three specifications: random walk (RW), mean-reverting process (MR), and random coefficient of the beta parameter (RC) for companies listed on NYSE and NASDAQ in the period 1990–2021. We examine the prognostic power of beta estimates and ranked the results according to criteria of forecast accuracy. In terms of the adopted criteria, the estimation of the beta parameter assuming its variability in time proved to be better than the OLS, LAD and ROLS methods of the Sharpe model. We can conclude that the Kalman filter approach with the assumption of a random coefficient (RC) or mean-reversion (MR) for the CAPM beta parameter gives the best results.

Keywords: rate of return, beta parameter, time-varying model, Kalman filter, US stock market

JEL: G10, G11, C22

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1. Introduction

The problem of proper CAPM beta estimation to measure systematic risk is important both for scientific studies and financial market practitioners. The estimates of systematic risk have different applications in finance, but they are especially used in Modern Portfolio Theory.[1] Simply, beta measures the risk associated with the stocks and investors can decide between the risky assets – the premium for one risky asset with reference to another (pricing risky securities). Additionally, CAPM beta is a framework for quantifying cost of equity when calculating Weighted Average Cost of Capital (WACC) and finally, portfolio beta is used in Treynor ratio – a risk-adjusted measurement of return based on systematic risk. Beta provides some information about risk and it is crucial for all investors and portfolio managers. There is little empirical work on the study of beta parameter nowadays, which appears even more striking when compared to the wide coverage of such research in the late 1990s.

To calculate the CAPM beta parameter, Sharpe's Single Index Model (1964) is used. This model is based on the assumption that stocks vary together because of the common movement in the stock market. This model assumes that there is only one factor affecting all stock returns (systematic risk) and this factor can be represented by the stock market index:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it} \tag{1}$$

There are no effects beyond the market that accounts for co-movement between stocks. Beta parameter measures the degree to which the volatility of a stock correlates to that of the market index.

Along these lines, our main motives for preparing this article were: (1) to develop the Kalman filter approach for estimating a beta parameter that is variable in time over the long time horizon (such studies have not been done yet), (2) to check whether beta depends on cyclical fluctuations in an economy determining the performance of systematic risk; therefore, our study includes data from the entire business cycle: economic prosperity and financial turmoil of the last 31 years, (3) to compare the results of different time-varying beta specifications based on the assumed stochastic structure of beta for US blue-chip companies.

The main purpose of this paper is to empirically examine the Kalman filter approach to estimating the time-varying beta parameter as a systematic investment risk in the most developed market in the world, the United States. The American market serves as a proxy for other developed countries around the world, and the study will result in more precise betas that have a significant effect on research findings. Some other research in the field of finance, especially related to modern portfolio theory, shows that this approach is usually the best for time-varying beta analysis and outperforms estimates obtained based on Sharpe's model. Therefore, the research question is to empirically check whether the Kalman filter approach to estimating the time-varying beta is statistically better and has greater predictive power than Sharpe's linear model. Three specifications of time-varying beta for the major US companies listed on the New York Stock Exchange and NASDAQ were explored. The research sample was built on the largest and most advanced stock market during the world in the period 1990 to 2021. The chosen companies belong to the S&P500 stock index and we collected only entire time series dataset.

[1] Capital Asset Pricing Model (CAPM) beta is estimated by the return of an asset based on the return of the market and the asset's linear relationship to the return of the market. It is estimated by the use of Sharpe's Single Index Model.

The results are compared with the estimates of the beta parameter obtained using Sharpe's linear model, which is the benchmark in our exploration. The estimations are made using the maximum likelihood method for monthly data. In our investigation, we rank the specifications used according to three criteria of fit and the matrix of correlation coefficients between the results of these specifications. The results of our study clearly show that the Kalman filter estimators bring better results when estimating the beta parameters than the linear Sharpe model. It demonstrates empirically that the best method used for beta estimation in long time horizon is the Kalman filter approach.

The Kalman filter approach may be a desirable supplement to standard econometric techniques, such as the Ordinary Least Squared (OLS) method, for tracking structural changes in systematic investment risk and estimating the beta parameter in time-varying markets. The Kalman filter beta estimates seem to be more appropriate for constructing a portfolio with minimum risk, and they are more powerful for long-term forecasting. To the best of our knowledge, there are not many studies that focus on the stochastic process of beta variation made on US stock exchanges. This research is the first that is primarily focused on such a large research sample of American companies in the last thirty-one years. These results could be valuable for all market participants, not only American investors, but all sorts of investment entities in the world economy where innovation is implicitly given by the research results. This research is an in-depth investigation focused on beta estimation and it is extremely valuable for financial analysts and researchers. The significance of the study presents the coverage of beta estimation techniques and proves the importance of the Kalman filter approach in systematic risk assessment during the long time horizon, including times of financial distress when beta vary over time (Global Financial Crisis of 2007–2009 and economic crisis caused by COVID-19 pandemic). This is a debatable topic in the literature of portfolio analysis, when investors and researchers have to properly quantify systematic risk. The novelty of this research is to utilize the Kalman filter approach in CAPM beta estimation to contribute more towards better systematic risk assessment in finance.

2. Literature review

The financial literature shows that there have been quite a number of techniques for beta estimation based on Sharpe's (1964) single-index model. When using the single-index model, it is still common practice to predict that the beta parameter will be invariant over time. The most basic approach to estimate beta is to estimate covariances and variances from a time series of historical stock returns. Nonetheless, this approach faces the problem of time-varying beta coefficients, e.g. Blume (1975), Ferson and Harvey (1991; 1993). The beta parameter might be unstable for many reasons; for example, the company might be changing its strategy, or the capital structure or returns might be affected by market movements or microeconomic factors, such as a change in the company's dividend policy, or a change of financial leverage. Such changes might cause beta variability over time. Many empirical studies from various financial markets have proved this, e.g. the American market (Fabozzi, Francis 1978), or the European market (Wells 1994; Chauveau, Maillet 1998). American stocks listed on NYSE have less than half of their total risk explained by market forces, thus it is true beta can move randomly while the OLS beta is invariant over the time period. The beta parameter for stocks listed on the Stockholm exchange is non-stationary and the research results justified its variability over time. Therefore, new methods for estimating the Sharpe model that assume the variability of the beta parameter have been proposed. One such method is the Kalman (1960) filter approach. Its basic advantage is the possibility of taking into account the stochastic variability of the beta parameter. The Kalman filters approach has been used as an estimation tool in modern portfolio theory, usually to assess the systematic risk measured by the beta parameter. It is an important econometric tool to estimate financial and economic problems connected with time-varying data in all financial markets worldwide. Kalman filters are not only applied to Sharpe's model, but also to stock price predictions on the spot and futures stock markets.

Berardi, Corradin, and Sommacampagna (2002) found an alternate approach to value-at-risk, estimating the betas of the assets in a portfolio with the Kalman filter with data from Nasdaq for the period 1999–2001. The results proved that the Kalman filter approach responded to market volatility changes and gave compelling results, thus this technique can capture the dynamics of financial markets in periods of high volatility and is flexible enough to match the hedging strategies of different financial institutions. Meanwhile, Gastadi and Nardecchia (2003) estimated the time-varying Italian industry beta parameter using the Kalman filter technique for the period 1991–2001. They stated that it is possible to estimate conditional time-dependent betas applying the Kalman filter to a sample of Italian industry portfolios and the results are more accurate when considering aggressive, highly leveraged companies or companies whose performance is unrelated to the general market behaviour. A similar observation was made by Frazzini and Pedersen (2014), who studied US and international equities from 20 countries and showed empirically that portfolios of high-beta assets have lower alphas and Sharpe ratios than portfolios of low-beta assets. International investors' behaviour of choosing high-beta financial assets indicates that the security market line for US stocks is too flat relative to the CAPM (Black, Jensen, Scholes 1972). This study poses the question of the asset pricing effect achieved by CAPM and sheds new light on the relation between risk and expected returns, questioning the CAPM beta explaining stock returns.

Meanwhile, Ebner and Neumann (2005) estimated the time-varying betas of 48 German stock returns. They used three estimation approaches – the Flexible Least Squares method, the Random Walk Model, and the Moving Window Least Squares. They strongly rejected the traditional market model with strong evidence of beta instability highlighting the efficiency of time-varying beta estimation in financial management.

Some research was also performed on the European market by Mergner and Bulla (2008). They investigated time-varying beta parameters estimated for 18 pan-European sectors in the period 1987–2005 using weekly data. The results showed that the random walk process, in conjunction with the Kalman filter, was the best at describing and forecasting time-varying sector betas in a European background. The authors stated that KF could be further improved by optimising beta forecasts and better explain the time-varying behaviour of systematic risk. Choudhry and Wu (2009) forecasted weekly time-varying beta using four different GARCH models and the Kalman filter method for data obtained from 1989 to 2003 for 20 British companies. The result was in favour of the Kalman filter approach, compared to the GARCH models – the Kalman filter dominates all GARCH models used in beta estimation.

The main reason for applying the Kalman filter to estimate systematic risk beta is that the state noise covariance and measurement noise covariance will be known. The techniques used in this case are widely known as the Adaptive Kalman Filter (AKF) (Martinelli 1995). Some applications were also made on the Indian securities market (Das, Ghoshal 2010; Das 2016). They applied the AKF with a modification for beta estimation using daily empirical data from the Indian market. The examination showed that the modified AKF could estimate the systematic risk measured by the beta parameter with good results. Later, Das (2016) presented a new formulation of a noise covariance adaptation based on second and third order AKF for joint estimation of two factor CAPM alpha and beta parameters. This research revealed that the higher order AKFs work equally as the Kalman filter approach, despite the flexibility in the time varying noise covariance (Das 2016).

Nieto, Orbe, and Zarraga (2014) researched the Mexican stock market from 2003 to 2009, mainly because of the high beta dispersion. They compared three methodologies to estimate time-varying betas: a rolling window OLS, multivariate GARCH (MGARCH) models, and the Kalman filter. The results showed that Kalman filter estimators with random coefficients outperform the others. An interesting application of the Kalman filter technique was employed to capture the time-varying degree of market integration change in global risk premium by Adam and Gyamfi (2015) on African stock markets. They showed that African financial markets are fully integrated into global markets and that the level of integration increased after the Global Financial Crisis, limiting the diversification opportunities. Hollstein and Prokopczuk (2016) made an interesting comparison of market beta estimation techniques on the constituents of the Standard & Poor index. They used GARCH models and Kalman filters among a wide range of approaches to estimate market beta. The Kalman filter approach with a random walk parametrization performed well in comparison with the GARCH models, which produced large errors. One of the latest studies was carried out by French (2016), who compared CAPM betas using a time--varying beta and a traditional constant beta model for five ASEAN countries and US sectors. Similar research was conducted by Tsuji (2017) on international CAPM time-varying betas for Asia Pacific and Japanese stock returns, and by Elshqirat and Sharifzadeh (2018) on the Jordanian Stock Market.

One of the most recent studies was carried out by Cisse et al. (2019) on the West African stock market. They compared two dynamics: one by the Kalman filter, assuming that the beta parameter follows a random walk, and the other by the Markov switching model, assuming that beta varies according to regimes. The results showed that the estimation by the Kalman filter fits better than the Markov switching model. Other similar methods were also used earlier on Central and Eastern European markets, e.g. Rockinger and Urga (2001). Meanwhile, Das (2019) proposed techniques for beta and VaR estimation of assets using adaptive Kalman filters based on National Stock Exchange of India indices. The results showed that sector betas are not constant but time-varying, and that modified adaptive Kalman filter techniques with unknown process and observation noise covariances perform at least as well as, or even better than, the traditional Kalman filters. Interesting studies were made by Aziz and Wibowo (2020) investigating the various approaches to model time-varying systematic risk in emerging markets of Indonesia and Thailand. They used GARCH, Schwert-Seguin,[2] and the Kalman filters to empirically find the most optimal time-varying beta estimation technique. The results showed that GARCH in Indonesia and TARCH[3] in Thailand outperform other models of beta estimation. Similar studies were made by Dębski, Feder-Sempach and Szczepocki (2020) to estimate the time-varying CAPM beta parameter in Poland, the Czech Republic, and Hungary. The results show that the Kalman filter estimators outperform the others. Asgar and Badhani (2021) showed low-beta-anomaly in the Indian equity market. They found a non-linear relationship between CAPM beta and expected returns and such a relationship follows a quadratic function. This means that stock returns initially increase

[2] Compare with Schwert and Seguin (1990).

[3] Threshold ARCH (TARCH).

with beta and then decline and that indicates the negative risk premium for high-beta portfolios. When estimating systematic risk in the context of emerging market economies of Europe and the Middle East, the Kalman filter-based wavelet approach was adopted by Asafo-Adjei et al. (2022). One of the outcomes from the correlations study showed that the degree of co-movements among the equity betas varies across countries and regions. Estimated beta parameters increased for most subregions and the emerging markets as a whole, implying rising market interconnection as the investment horizon was prolonged, which proved strong interdependence between countries. The Kalman filter-based wavelet multiple approach examined the degree of interdependence in the systematic risk returns measured by beta, implying some risk management solutions in portfolio diversification. This study can potentially explain Kalman filter equity beta's regional co-movements of systematic risk experiencing the stock market shocks among the selected emerging markets.

3. Research sample

The American stock market takes up about 50% of the global stock market structure. The United States plays a unique role in the global financial sector, not only as the world's largest financial market, but also as a global financial hub. The US stock market is the largest; it is very liquid, deep, and developed, with a long price history (Miziołek, Feder-Sempach, Zaremba 2020). The sample was selected based on representative criteria in terms of both the role of major American companies in the world's economy and market capitalization.

The research sample consists of 239 American companies listed on the New York Stock Exchange (NYSE) and NASDAQ. These companies belong to the S&P500 blue-chip stock index. We collected the closing prices of 239 stocks and the S&P500 index beginning from the last trading day of December 1989 to the last trading day of June 2021, which gives 379 monthly observations. We took all the companies with full time series available in the Refinitiv EIKON database.

Based on these prices, the logarithmic monthly rates of return were calculated as $R_{it} = (\ln P_{it} - \ln P_{it-1}) \times 100$, where R_{it} is the logarithmic rate of return on the i-th share at time t, and P_{it} is the price of the i-th share at time t. The rates of return on shares were calculated without dividends. For each company, we received 378 observations of return in the period 1990–2021 (31 years). We used monthly data because the vast majority of studies on the Kalman filter approach use this data interval due to the higher probability of normal distribution of the tested rates of return on shares. Data were obtained from the Refinitv EIKON database, and all tables are labelled with the RIC (Refinitv Instrument Code). They formed the basis for the present research. Figure 1 presents the time series of the S&P500 index and monthly log returns for the considered period. We see high volatility of returns over time, which also suggests a large variability of the beta parameter over time.

We calculated typical descriptive statistics for the time series of the logarithmic rates of return for each company from our database. Table 1 presents means across the individual statistics. We see that the mean skewness value is slightly negative (-0.56), and the mean kurtosis value is positive and quite high (8.502). This means that most of the surveyed companies have a left-skewed distribution of return, and it is highly concentrated around the mean value.

4. Research methodology

In the study, we used the following approaches to estimate the beta parameter of Sharpe's Single Index Model (Sharpe's model) with an invariant or variant beta parameter: Ordinary Least Squares (OLS), Least Absolute Deviations (LAD), Rolling Ordinary Least Squares (ROLS), and the Kalman Filter (KF) approach for three different specifications of the beta parameter, see Figure 2.

4.1. Ordinary least squares

The first approach is the original Sharpe model with invariant beta parameter:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}, \qquad \varepsilon_{it} \sim \text{IID}N\left(0, \sigma_{i\varepsilon}^2\right), \qquad i = 1, \dots, I, \;\; t = 1, \dots, T \tag{2}$$

where:

R_{it} – logarithmic rate of return of the i-th share (i – 1,…, I) at time t,
R_{Mt} – logarithmic rate of return of the S&P500 index at time t,
ε_{it} – random term at time t.

The unknown coefficients (α_i, β_i, $\sigma_{i\varepsilon}^2$) in (1) are estimated by OLS. The model was developed by William Sharpe (Sharpe 1963).

4.2. Least Absolute Deviations

The second approach is to estimate equation (2) with Least Absolute Deviations (LAD). This statistical optimization technique is based on minimizing the sum of absolute deviations:

$$\left(\hat{\alpha}_{it}, \hat{\beta}_{it}\right)_{LAD} = \underset{\alpha_{it}, \beta_{it}}{\arg\min} \sum_{i=1}^{t} \left| R_{it} - \alpha_{it} - \beta_{it} R_{Mt} \right| \quad i = 1, \dots, I, \;\; t = 1, \dots, T \tag{3}$$

The Ordinary Least Squares method of estimation of parameters of linear regression models performs well, provided that the residuals are well behaved. However, models with the residuals that are non-normally distributed and contain sizeable outliers fail estimation by the Ordinary Least Squares method. This sensitivity is due to the quadratic loss function. Least Absolute Deviations based on absolute loss function are more robust against outliers. LAD regression is equivalent to quantile regression with τ = 0.5 (median) and maximum likelihood estimation of linear regression model if the errors have a Laplace distribution.

4.3. Rolling Ordinary Least Squares

The third approach is to estimate betas by a Rolling OLS estimation of the original Sharpe model. The model was proposed by Fama and MacBeth (1973). This method applies OLS across a fixed window of observations and then rolls the window across the data set. For each window, a local sum of squared residuals is minimized:

$$\left(\hat{\alpha}_{it}, \hat{\beta}_{it}\right)_{ROLS} = \underset{\alpha_{it}, \beta_{it}}{\arg\min} \sum_{j=t-r+1}^{t} \left(R_{it} - \alpha_{it} - \beta_{it} R_{Mt}\right)^2 \quad i = 1, \ldots, I, \ t = r, \ldots, T \tag{4}$$

In the research, we used a window of 36 monthly observations.

4.4. The Kalman filter approach

The last approach for estimating the time-varying beta parameter is to re-write Sharpe's model into the form of a linear Gaussian state-space representation, and then use the Kalman filter. The state--space form consists of two equations: a measurement equation and a transition equation. The former is Sharpe's model with constant alpha parameter and time-varying beta parameter:

$$R_{it} = \alpha_i + \beta_{it} R_{Mt} + \varepsilon_{it}, \qquad \varepsilon_{it} \sim \text{IID}N\left(0, \sigma_{i\varepsilon}^2\right), \quad i = 1, \ldots, I, \ t = 1, \ldots, T \tag{5}$$

where $\sigma_{i\varepsilon}^2$ is the variance of the measurement error.

The transition equation specifies the form of the stochastic process that the beta parameters are assumed to follow. We consider three different representations of the transition equation: a random walk of the beta parameter (RW), a mean-reverting process for the beta parameter (MR), and a random coefficient of the beta parameter (RC).

In the first specification (RW), the beta parameter should follow the Gaussian random walk process:

$$\beta_{it} = \beta_{it-1} + \nu_{it}, \qquad \nu_{it} \sim \text{IID}N\left(0, \sigma_{i\nu}^2\right), \quad i = 1, \ldots, I, \ t = 1, \ldots, T \tag{6}$$

The variance $\sigma_{i\nu}^2$ of state error terms ν_{it} determines the stability of the time-varying beta. When this coefficient decreases, less variability is allowed, and the time-invariant beta parameter becomes more stable. In a limiting scenario, when the variance $\sigma_{i\nu}^2$ tends to zero, the original Sharpe model is also obtained. The random walk of the beta parameter is considered by many previous studies, e.g. Faff et al. (2000), Ebner and Neumann (2005), Choudhry and Wu (2008), Das and Ghoshal (2010), Kurach and Stelmach (2014), and Będowska-Sójka (2017).

The second specification (MR) of the transition equation is the mean-reverting process for the beta parameter with a long-term mean β_i and autoregressive parameter ρ_i ($|\rho_i| < 1$):

$$\beta_{it} = \beta_i + \rho_i\left(\beta_{it-1} - \beta_i\right) + \nu_{it}, \quad \nu_{it} \sim \text{IID}N\left(0, \sigma_{i\nu}^2\right), \quad i = 1, \ldots, I, \ t = 1, \ldots, T \tag{7}$$

The autoregressive parameter determines the strength of the mean reversion toward the long-term mean, and the higher the parameter value, the longer the shock persists. The state representation with this type of transition equation was studied by Yao and Gao (2004), Jostova and Philipov (2005), and Kurach and Stelmach (2014).

The last specification of the transition equation, the random coefficient beta parameter (RC), may be considered a special case of the mean-reverting process, when the autoregressive parameter ρ_i is equal to zero:

$$\beta_{it} = \beta_i + \nu_{it}, \qquad \nu_{it} \sim \text{IID}N\left(0, \sigma_{i\nu}^2\right), \quad i = 1, \ldots, I, \; t = 1, \ldots, T \tag{8}$$

In the random coefficient assumption, the beta parameter is assumed to vary randomly around a fixed value β_i without any persistence. The variance $\sigma_{i\nu}^2$ controls the variations of the beta parameter. The state representation with this transition equation was applied by Faff et al. (2000), Yao and Gao (2004), and Ebner and Neumann (2005).

In the Kalman filter approach, we estimated the parameters of the state space representations using the maximum likelihood method. As the set of the initial value for the state vector and covariance matrix, we used the OLS estimates from the entire sample.

5. Discussion of results

Next, we wanted to check the predictive power of the beta parameter. We split the entire period of returns into two parts: in-sample and out-of-sample. The former embraces 258 observations from January 1990 to June 2011, and the latter, the remaining 120 observations from July 2011 to June 2021. This split point was chosen ad-hoc to get a ratio close to 80%/20%. The in-sample period includes data from entire business cycle, both periods of economic prosperity and financial crises (e.g. dot-com bubble, subprime mortgage crisis and COVID crisis). There are no broadly accepted guidelines for how to select the sample split (Hansen, Timmermann 2012). We estimated the four models (OLS, QR, ROLS, KF RW, KF MR, KF RC) introduced earlier, only on the basis of the in-sample period.

Firstly we calculated two measures of in-sample goodness of fit:

1) Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(R_{it} - \hat{R}_{it}\right)^2}$$

2) Mean Absolute Error (MAE):

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|R_{it} - \hat{R}_{it}\right|$$

where $\hat{R}_{it}$ $(t = 1, \ldots, T)$ is the fitted value of the logarithmic rate of return on the *i*-th share at time *t*. The summary results are presented in Table 2.

As we can see, estimating the beta parameter using the Kalman filter approach with three assumptions about its variability gave the best results. They outperform the estimation results obtained by applying the ROLS methods to the Sharpe model. Of the three specifications of the time-varying beta parameter adopted, the one with the RC assumption proved to be the best. This means that it gives the smallest forecast errors, i.e. the estimated beta parameter has the greatest predictive power in-sample. The worst results were obtained by the ROLS estimation of the linear Sharpe model. Using the Kalman filter approach produced results that were much better.

Furthermore, we calculated the out-of-sample forecast of the beta parameter. For OLS and LAD, we simply took the parameter estimates for all companies from the in-sample period to the out-of-sample period. For ROLS, we took the parameters estimates from the last window containing time period t and remained unchanged for time period $t + 1$. For the Kalman filter specifications (RW, MR, RC), we took the estimated parameters from the in-sample period and then filtered the out-of-sample period with the initial values from the last observations from the in-sample period.

Finally, we calculated two measures of out-of-sample forecast errors (RMSE, MAE). The summary results are presented in Table 3 and more detailed in the appendix. We then repeated the out-of-sample forecast with a re-estimation of the OLS, LAD and Kalman filter specifications. Table 4 presents the summary results with re-estimation after five years, Table 5 after two years, and Table 6 after one year. The forecast errors of ROLS are the same in Tables 3–6, because re-estimation of the model was done every month during the whole out-of-sample period.

As we can see in the Table 3, two measures of out-of-sample forecast errors (RMSE MAE) for the models with Kalman filter RC gave the best results.

The results in Tables 3–6 clearly indicate that the Kalman filter approach with the assumption of a random coefficient (RC) process or mean-reversion (MR) for the beta parameter gave better results in forecasting the rates of return of the 239 surveyed companies than the other specifications of the beta parameter used in our article. The beta parameter estimator from these specifications has the greatest prognostic power (brings smaller forecast errors), but it does not matter whether it is a re-estimation after one, two or five years. This is indicated by all the criteria used to measure forecast errors. The worst results of the beta parameter estimation in terms of its prognostic power were given by the ROLS method.

Summing up the above results, we can conclude that estimating the beta parameter using the Kalman filters and taking into account its variability in time is more effective in terms of forecast errors than the estimation of the linear Sharpe model using the OLS, LAD or ROLS methods. The results proved that when estimating the beta parameter and the subsequent forecasting based on it, it is worth taking into account the variability of this parameter. First of all, time-varying beta should be specified on the basis of the mean-reverting process and random walk process depending on reestimation frequency.

6. Summary

The main aim of the article was to check the statistical effectiveness of estimating the beta parameter assuming its variability in time using the Kalman filter approach, and then to compare it with the OLS, LAD and ROLS estimations of the linear Sharpe model using the world's leading stock exchange data. We wanted to demonstrate this through appropriate information criteria and testing the predictive power of the beta parameter. We purposely selected companies with the largest market capitalization from the most developed market – the world's financial hub, NYSE and NASDAQ. The number of surveyed companies was relatively large (239), and the length of the time series of rates of return was relatively long including the entire business cycle: economic prosperity and the financial turmoil of 2007–2009 and COVID-19 recession (monthly data for 31 years). We did it so that the results of the study could be generalized as far as possible. In terms of prognostic power, the estimation of the beta parameter assuming its variability in time (the specification with mean reverting process) proved to be better than the estimation of this parameter using the OLS and ROLS methods of the Sharpe model.

Also, in terms of the predictive power of the beta parameter, at least two specifications of its variability over time, i.e. the KF RC and KF MR specifications, outperformed the results of the OLS, LAD and ROLS forecasts. The calculated out-of-sample forecast errors were smaller for these two specifications of the time-varying beta parameter. Based on data obtained from the two major American stock exchanges in the last thirty-one years, we can conclude that the Kalman filter is the optimal tool for beta estimation. We find robust empirical evidence for the presence of the Kalman filter in the long time horizon with different business cycles. Therefore, the Kalman filter can be fully competitive and it leads to more accurate risk estimation over the long time horizon. This gives rise to a legitimate hope of using the specification with a time-varying beta parameter in other highly developed markets, e.g. the euro zone or the United Kingdom, or perhaps emerging economies.

This study is limited to US market data and only three specifications of time-varying beta – random walk (RW), mean-reverting process (MR) and random coefficient of the beta parameter (RC). The results presented in this article advocate further research in this field, applying different financial markets, especially advanced European economies, longer time periods including high volatility, i.e. the entire COVID-19 economic crisis and more modern methods like LSTM (Long Short-term Memory Networks). In turn, more effective beta parameter estimates, as well as their greater prognostic power, can be used to more accurately estimate investment risk, which will certainly interest all investors who build and manage investment portfolios. A more precisely estimated investment risk is highly desirable and effective in financial market analysis and allows the achievement of better returns on investments. Beta is crucial when it comes to portfolio management: perhaps it is the most important measure of a stock's risk in finance. Through this research, the importance of the Kalman filter approach in systematic risk assessment during times of financial distress when beta can vary over time is elucidated.

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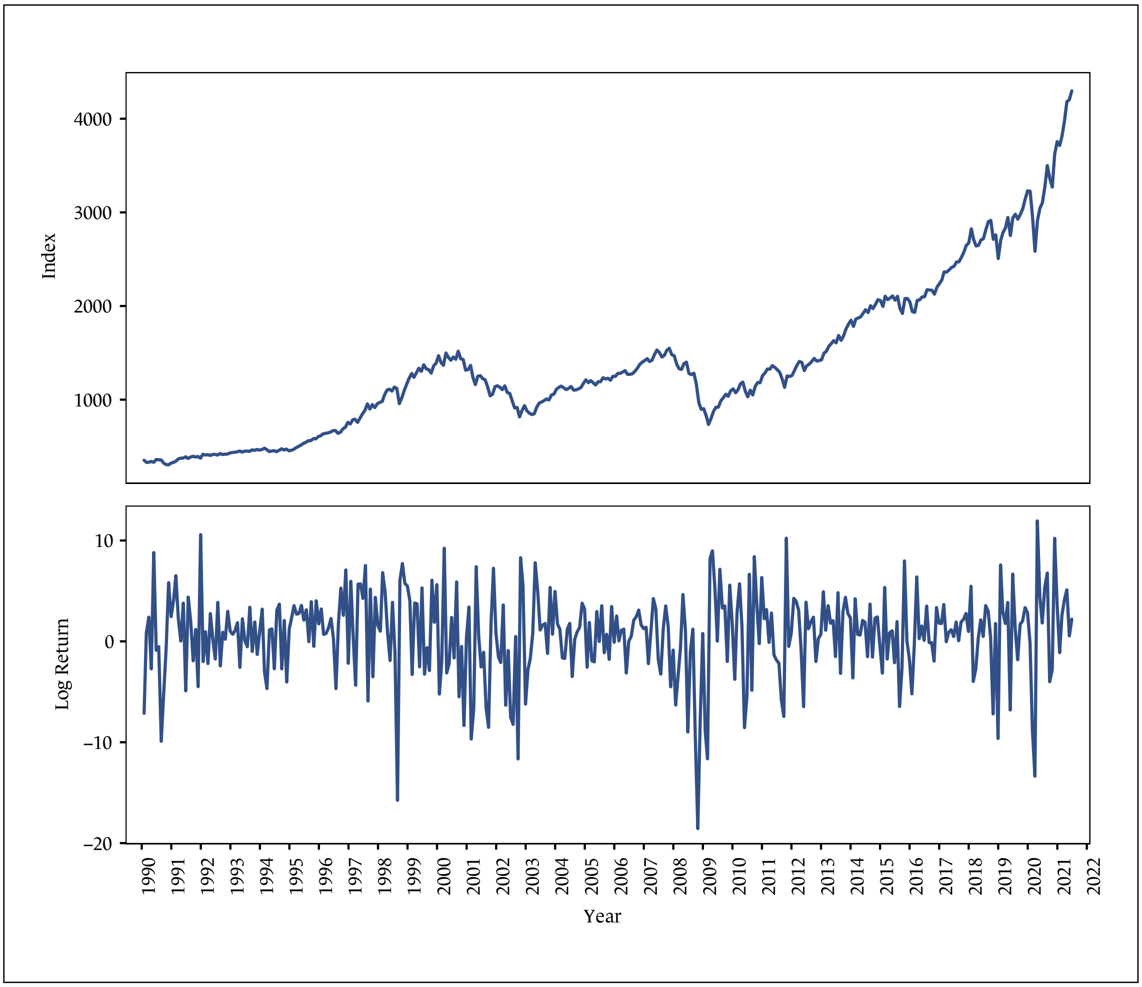
Appendix

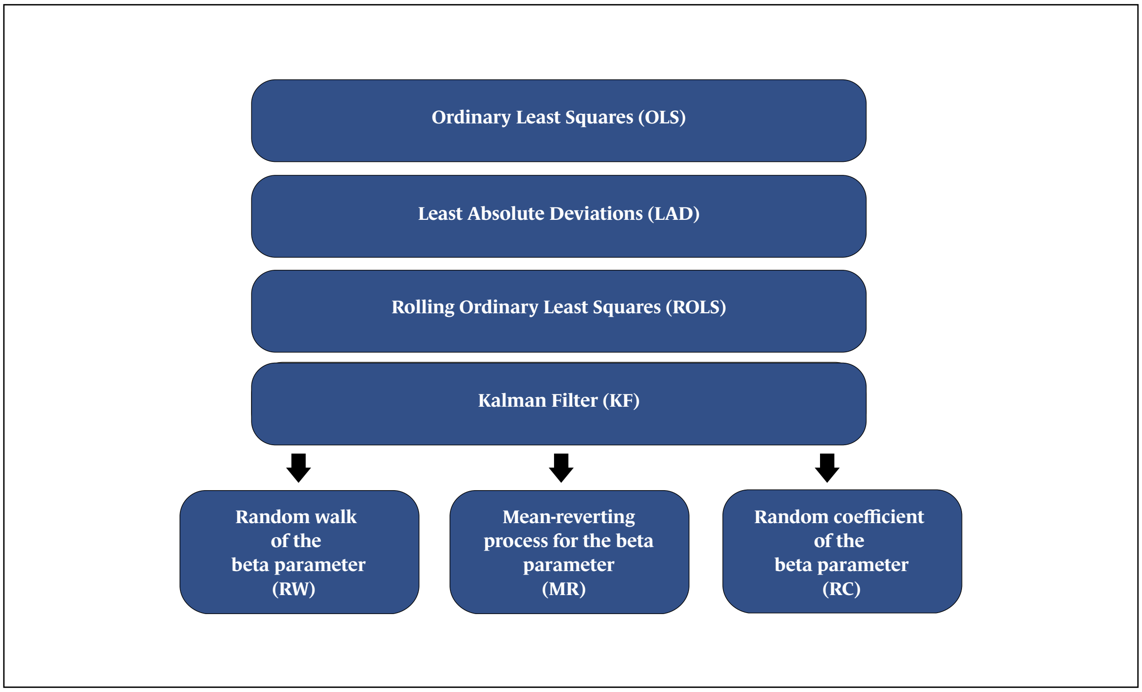
Figure 1

The time series of S&P500 index and monthly log returns from January 1990 to June 2021



Source: own research based on Refinitiv EIKON data.

Figure 2
Beta estimation approaches used in the research



Source: own research.

Figure 3
Out-of sample forecast errors measured by RMSE

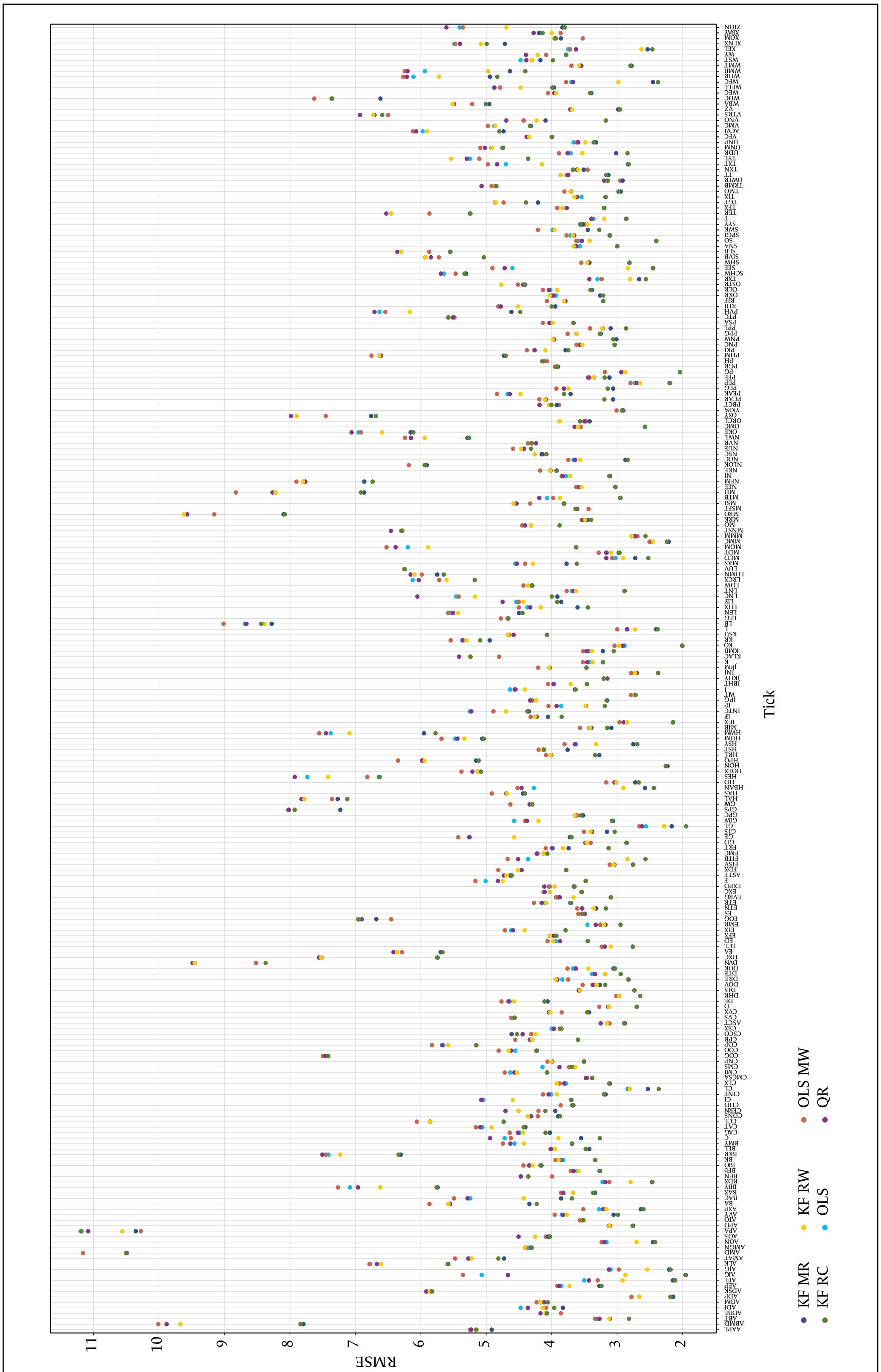


Figure 4
Out-of sample forecast errors measured by MAE

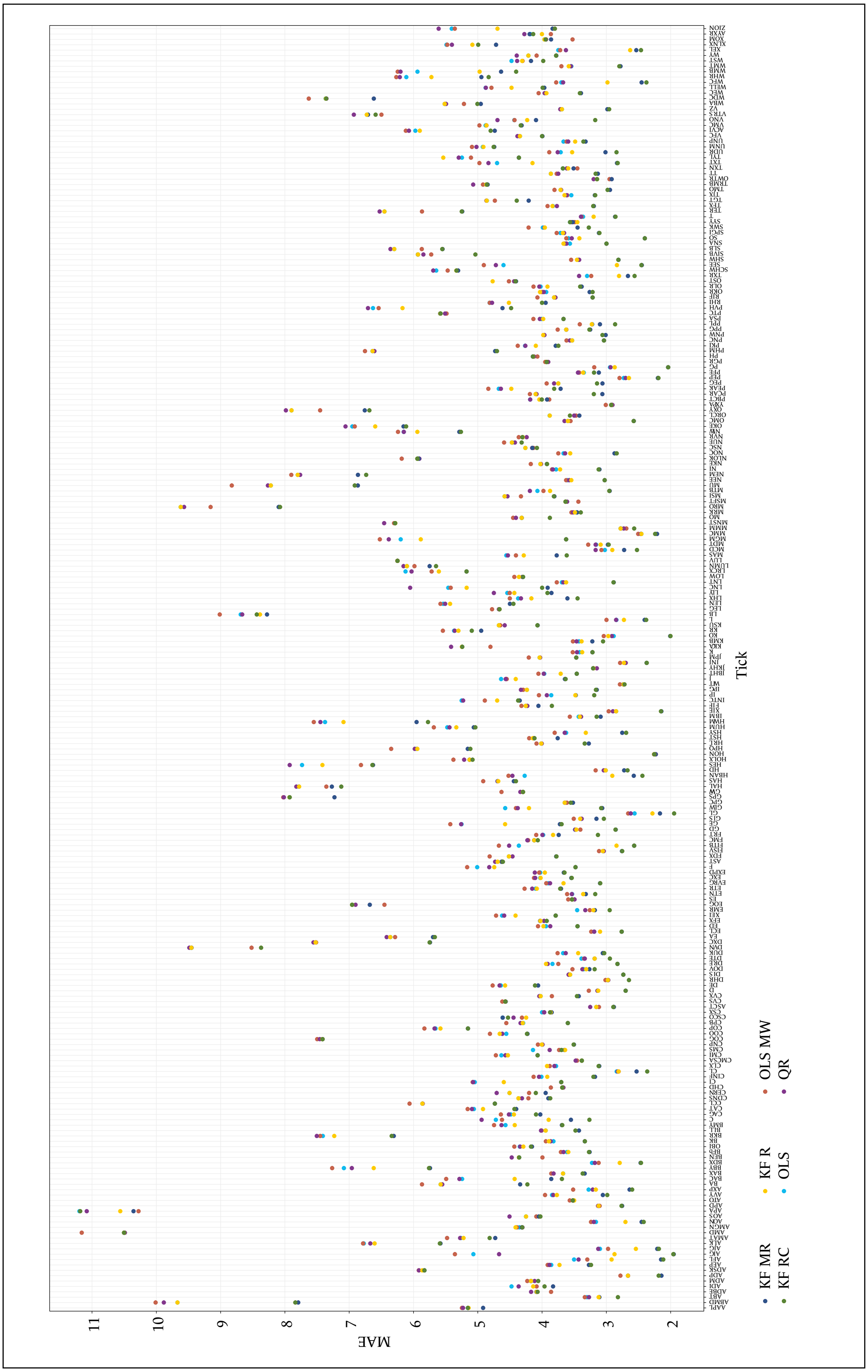


Table 1
Means of descriptive statistics calculated across all 239 series of stocks and S&P 500 index logarithmic rates of return used in the study

Mean	Standard devation	Skew-ness	Kurtosis	Mini-mum	1st quartile	Median	3rd quartile	Maxi-mum
0.779	8.876	-0.56	8.502	-45.404	-3.812	1.128	5.803	36.29

Source: own research.

Table 2
Rankings of measures of in-sample forecast errors based on the sum of the ranks across companies, with the sum of the ranks in parentheses

Goodness of fit criteria	Method of estimation					
	OLS	ROLS	LAD	KF MR	KF RW	KF RC
RMSE	8.295	9.033	8.325	6.984	7.834	6.95
MAE	6.013	6.539	5.986	5.12	5.743	5.095

Source: own research.

Table 3
Means of out-of sample forecast errors (models without re-estimation)

Forecast criteria	Method of estimation					
	OLS	ROLS	QR	KF MR	KF RW	KF RC
RMSE	6.102	6.097	6.136	5.256	5.884	5.237
MAE	4.568	4.589	4.584	3.976	4.434	3.956

Source: own research.

Table 4
Means of out-of sample forecast errors (re-estimation every five years)

Forecast criteria	Method of estimation					
	OLS	ROLS	QR	KF MR	KF RW	KF RC
RMSE	6.085	6.097	6.11	5.217	5.856	5.174
MAE	4.556	4.589	4.568	3.956	4.426	3.922

Source: own research.

Table 5

Means of out-of sample forecast errors (re-estimation every two years)

Forecast criteria	Method of estimation					
	OLS	ROLS	QR	KF MR	KF RW	KF RC
RMSE	6.074	6.097	6.096	5.201	5.848	5.163
MAE	4.549	4.589	4.558	3.939	4.422	3.912

Source: own research.

Table 6

Means of out-of sample forecast errors (re-estimation every year)

Forecast criteria	Method of estimation					
	OLS	ROLS	QR	KF MR	KF RW	KF RC
RMSE	6.071	6.097	6.093	5.185	5.851	5.146
MAE	4.547	4.589	4.555	3.927	4.426	3.901

Source: own research.

Parametr beta zmienny w czasie. Zastosowanie filtrów Kalmana do oceny ryzyka systematycznego amerykańskich spółek w długim horyzoncie czasowym

Streszczenie

Głównym celem artykułu jest zastosowanie filtrów Kalmana do oceny ryzyka systematycznego, mierzonego parametrem beta (CAPM), na amerykańskim rynku giełdowym – NYSE i NASDAQ – w długim horyzoncie czasowym. Okres badania obejmuje 31 lat, co oznacza przejście gospodarki przez wszystkie fazy cyklu koniunkturalnego: fazę ożywienia, rozkwitu oraz kryzysu i depresji, które powodują występowanie znacznych strat finansowych i społecznych (kryzys finansowy w latach 2007–2009 i kryzys COVID-19). Wahania cykliczne w gospodarce wpływają na stopy zwrotu uzyskiwane z inwestycji w akcje i są pochodną ryzyka systematycznego, tak ważnego w analizie opłacalności inwestowania. Badanie przeprowadzono na podstawie notowań amerykańskich spółek na NYSE i NASDAQ w latach 1990–2021 z miesięcznym interwałem czasowym pomiaru stopy zwrotu. Oszacowanie ryzyka systematycznego zostało przeprowadzone na podstawie trzech specyfikacji modelu zakładającego zmienność parametru beta w czasie. Były to: błądzenie losowe parametru beta (RW – *random walk*), proces zakładający powrót bety do średniej (MR– *mean-reverting process)* i współczynnik losowy parametru beta (RC – *random coefficient*). Wyniki zostały uszeregowane na podstawie mocy prognostycznej oszacowań beta i dwóch kryteriów dokładności prognozy. Pod względem przyjętych kryteriów estymacja parametru beta, przy założeniu jego zmienności w czasie, wykorzystująca filtr Kalmana okazała się lepsza niż klasyczne metody OLS, LAD i ROLS. Na podstawie otrzymanych wyników można jednoznacznie stwierdzić, że filtr Kalmana, zakładający współczynnik losowy parametru beta (RC) oraz powrót bety do średniej (MR), może być optymalnym narzędziem do estymacji ryzyka systematycznego na amerykańskim rynku giełdowym. Estymacja parametru beta przy wykorzystaniu filtrów Kalmana dla badanej próby okazała się lepsza pod względem dokładności i mocy predykcyjnej. Przeprowadzone badanie jasno wskazuje, że filtr Kalmana może być optymalnym narzędziem oceny ryzyka systematycznego w długim horyzoncie czasowym i może być wykorzystywany w skutecznej analizie ryzyka rynków finansowych w krajach rozwiniętych. Interesujące byłoby powtórzenie badania dla krajów rozwijających się i rozszerzenie próby badawczej o inne rynki o znaczeniu międzynarodowym, np. rynek brytyjski.

Słowa kluczowe: stopa zwrotu, parametr beta, beta zmienna w czasie, filtr Kalmana, amerykański rynek giełdowy