

# Mortgage down-payment and welfare in a life-cycle model

Michał Rubaszek\*

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## Abstract

The paper presents a standard incomplete markets life-cycle model with housing that is calibrated to the Polish economy. The model is used to analyze the effects of changes in the minimum down-payment restriction on welfare. Our analysis shows that tougher credit conditions have a significant impact on the level of household debt, the ownership ratio and the time of first house purchase. However, assuming well functioning rental market the welfare costs are not sizable.

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\* National Bank of Poland and Warsaw School of Economics; e-mail: [michal.rubaszek@nbp.pl](mailto:michal.rubaszek@nbp.pl).

## 1. Introduction

The traditional approach to housing market booms was one of “benign neglect”. This approach relied on the view that distortions associated with preventing a boom outweigh the costs of cleaning up after a bust. The recent recession, which was triggered by the subprime crisis in the U.S., has changed this view. For that reason macroprudential policy measures, among others a minimum down-payment restriction on mortgages, has become a highly debated topic among central banks and financial supervision authorities.<sup>1</sup> On the one hand, an introduction of a limit on the loan-to-value ratio reduces the risk that during a bust a large number of homeowners would find themselves “under the water”, i.e. in the situation in which their houses are worth less than mortgages. On the other, it would be welfare detrimental to the extent that some households would find themselves rationed out of the credit market, and will not be able to purchase a house.

The aim of this article is to evaluate long-term welfare costs associated with a change in regulations related to the minimum mortgage down-payment. These costs stem from the fact that an increase in the down-payment requirements make some households rationed out of the credit market. For that purpose we develop a standard incomplete market life-cycle model with housing. Households, which are subject to idiosyncratic income fluctuation, take decisions on consumption, savings, the size and ownership status of houses they live in. It should be noted that since in the model there is no default or aggregate risk, it is not well suited to analyze the positive effects of changes in the minimum down-payment restrictions on the stability of the financial system. The model is parameterized so that it was suitable to analyze the Polish economy. Finally, we use the model to analyze how changes in the minimum down-payment requirement affect the economy.

Our work is closely related to the classic income fluctuation problem, in which households decide how much to consume and save. The seminal contributions in a partial equilibrium framework include articles by Carroll (1997), Zeldes (1989) and Deaton (1991), whereas in a general equilibrium setup the articles by Huggett (1993; 1996) and Aiyagari (1994). Moreover, since we explicitly model the housing sector, our paper is closely related to several more recent works. The article of Fernandez-Villaverde and Krueger (2011), with the working paper version dating back to 2002, was the first to show that consumer durables are crucial to explain the life cycle profiles of consumption and savings. Their analysis was further extended by Yang (2009), who clarifies that borrowing constraints are essential in explaining the accumulation of housing stock early in life, where transaction costs are crucial in generating the slow downsizing of the housing stock later in life. The role of the minimum down-payment restriction in this class of models was extensively analyzed by Ortalo-Magne and Rady (2006), Iacoviello and Pavan (2009), Chambers, Garriga and Schlagenhauf (2009) and Bajari et al. (2010). Ortalo-Magne and Rady developed a life-cycle model illustrating the impact of housing regulations on house price volatility. Iacoviello and Pavan show that an increase in idiosyncratic income uncertainty combined with a decrease in the minimum down-payment requirement helped to reduce the volatility of GDP in the US in the period before the recent crisis. Chambers, Garriga and Schlagenhauf analyze to what extent

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<sup>1</sup> Rabanal et al. (2011) provide an excellent discussion on policy actions that can be taken to deal with real estate booms, including an extensive description of country experiences.

mortgage market innovations account for the observed increase in the homeownership rate in the United States. Finally, Bajari et al. simulate a dynamic life-cycle model to analyze the effects of a collapse in home prices and a tightening of credit standards experienced in the U.S. during the recent crisis.

Our study adds to the above literature, which uses life-cycle models with housing to analyze the functioning of the economy. The main contribution of this study is to explore welfare effects of changes to housing market regulations. Moreover, according to our knowledge this is the first article that calibrates this kind of model to match the Polish data.

The structure of the article is as follows. In Section 2 we present the structure of the model. Then, in Section 3 we introduce the concept of stationary equilibrium. Section 4 provides a discussion on the choice of parameters. Section 5, which is the main part of the article, presents the results. The last section concludes and discusses potential directions for further research.

## 2. The model

We consider a life-cycle economy with uncertain lifespan and idiosyncratic productivity. Households derive utility from consumption of non-housing goods and housing services, as well as from leaving bequests. They prefer owning a house to renting it, but are constrained in purchasing the house because of the minimum down-payment requirement. In this economy there are four reasons why households accumulate savings: to finance consumption after retirement, to self-insure against unexpected individual income shocks, to leave bequests, and to accumulate assets for mortgage down-payment. Finally, it can be noted that in the model housing plays a dual role: as a consumption good and as an investment asset. The structure of the model is as follows.

### 2.1. International capital markets

The model economy is small and open with access to international capital markets. The level of the domestic real interest rate is

$$r = r^* - \xi \frac{B}{Y} \quad (1)$$

where  $r^*$  stands for the foreign real interest rate,  $B$  denotes the value of net foreign assets, and  $Y$  is the level of domestic output. The parameter  $\xi$  measures the level of international financial markets imperfections. Two special cases are autarky, in which  $\xi \rightarrow \infty$ , and perfect international financial markets, in which  $\xi = 0$  (see Ca' Zorzi, Rubaszek 2012 for extended discussion).

### 2.2. Firms

The goods market is perfectly competitive and characterized by constant returns to scale. Identical firms of measure one are producing a homogeneous final good according to the Cobb-Douglas technology.

$$Y = K^\alpha L^{1-\alpha} \quad (2)$$

where  $K$  and  $L$  denote the aggregate capital stock and effective labor input, respectively.

The final good can be consumed, invested in physical capital or transformed into housing. Factor prices are determined by profit maximization and are equal to their marginal products

$$\begin{aligned} \partial Y / \partial K &= r + \delta \\ \partial Y / \partial L &= w \end{aligned} \quad (3)$$

where  $w$  is the real wage and  $\delta$  stands for the depreciation rate.

### 2.3. Demographics and preferences

The economy is populated by an infinite number of households of different age  $j \in \mathfrak{A} \equiv \{1, 2, \dots, J\}$ . Their lifespan is uncertain: the probability of being alive next year at age  $j$  is equal to  $s_j$ . The unconditional probability of surviving till age  $j > 1$  at time of birth is  $S_j = \prod_{i=1}^{j-1} s_i$ . Finally, we normalize the size of total population to unity and denote the share of cohort  $j$  in population by  $\mu_j$ .

Households derive utility from consumption of non-housing goods  $c$  and the service flow of housing, which can be owned  $h_o$  or rented  $h_r$ . We assume that the utility of renting a house is lower than owning it, among others because renters are often restricted to modify the property they rent. The momentary utility function is of the form

$$u(c, h_o, h_r) = \frac{(c^\theta (\max\{h_o, \vartheta h_r, \varepsilon_h\})^{1-\theta})^{1-\eta}}{1-\eta} \quad (4)$$

Here  $\vartheta \leq 1$  measures the disutility of renting compared to owning,  $\theta$  is the share of non-housing consumption in the utility,  $\eta$  stands for the risk aversion, and  $\varepsilon_h$  is a small number that makes the utility function finite for  $h_o = h_r = 0$ .

Households derive additional utility

$$u_b(b) = \kappa \frac{b^{1-\eta}}{1-\eta} \quad (5)$$

from giving a bequest  $b$ . The degree of altruism is governed by the parameter  $\kappa$ . For  $\kappa = 0$  bequests are accidental, generated by the fact that the lifespan is uncertain. Finally, we assume that all bequests are collected by the government and equally distributed among households in the form of lump-sum transfers  $tr$ .

## 2.4. Individual income process

The economic activity of households consists of two distinct periods. During the initial  $\tilde{J}$  years every household works by supplying one unit of time in the labor market. The productivity is the product of age-dependent deterministic component  $z_j$  and a stochastic component  $e \in E \equiv \{e_1, e_2, \dots, e_K\}$ , i.e.  $z(e, j) = z_j \times e$ . The stochastic component follows a Markov process with the elements of the transition matrix  $\pi_{kl} = P(e' = e_l | e = e_k)$ , where  $\pi_{kl} > 0$  and  $\sum_{l=1}^K \pi_{kl} = 1$  for every  $k, l \in \{1, 2, \dots, K\}$ .<sup>2</sup> In the second part of their life households retire and receive a lump-sum pension  $pen$  that equals to a fraction  $\chi$  of the average gross wage  $\bar{w}$ . The resulting income from labor is

$$y(e, j) = \begin{cases} (1 - \tau)wz(e, j) & \text{for workers} \\ pen & \text{for retirees} \end{cases} \quad (6)$$

Here  $\tau$  stands for the social contribution rate, which is set so that the pension system budget is balanced

$$\tau wL = pen \sum_{j=\tilde{J}+1}^J \mu_j \quad (7)$$

## 2.5. Financial sector and the housing market

A perfectly competitive financial sector collects deposits from households, and grants loans to firms, landlords and for mortgages. It also participates in the international capital market, where it can raise funds at the interest rate given by (1). Since there is no risk in the model, interest rates on deposits and loans are the same.

Landlords specialize in renting the residential capital to households. At the end of period they need to spend a fraction  $\varphi$  of the house value on repair and maintenance to keep housing quality constant. The no-arbitrage condition implies that the rental price on housing equals to

$$p_r = r + \varphi \quad (8)$$

As regards homeowners, we consider two main frictions in the housing market. First, the stock of housing is subject to nonconvex adjustment costs. In particular, in order to change a house of size  $h$  to a house of size  $h'$  a household has to pay

$$\phi(h, h') = \begin{cases} \phi_1 h + \phi_2 h' & \text{if } h' \neq h \\ 0 & \text{if } h' = h \end{cases} \quad (9)$$

where  $\phi_1$  and  $\phi_2$  measure the transaction cost of selling and buying a house, respectively.

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<sup>2</sup> Throughout the paper we use  $x'$  to denote the next period value of a variable  $x$ .

The second friction states that buying a house requires an amount of cash up front. The down-payment equals to at least a fraction  $\gamma \geq 0$  of the house value, which means that in each period net financial position of a household  $a$  must satisfy

$$a \geq -(1 - \gamma)h \quad (10)$$

It can be noted that if  $\gamma$  is high, the minimum down-payment requirement limits the value of many first home purchases, primarily for younger households with little savings. Once the household owns its first home, a high level of the minimum down-payment might delay the purchase of a bigger house if the household wants to move up the property ladder. This is the main reason why an increase in the down-payment requirement is welfare detrimental.

Finally, we assume that the size of an owned house is a discrete variable with values from the set  $H_o \equiv \{0, h_1, h_2, \dots, h_M\}$ . As for the size of a rented house, it can take also an additional value  $h_0 < h_1$ , so that  $H_r \equiv h_0 \cup H_o$  (see e.g. Iacoviello, Pavan 2009).

## 2.6. Timing and the budget constraint

A household of age  $j$  enters a period with the housing stock  $h$  net of maintenance costs  $\phi$ , financial assets  $a$  augmented by the interest rate  $r$ , and with the idiosyncratic productivity  $e$ . Subsequently, she or he receives labor income  $y(e, j)$  and transfers  $tr$ , and decides how to divide the resources on consumption  $c$ , housing  $h'$  and  $h_r$ , and savings  $a'$ . Moreover, if she or he decides to change the size of the house the transaction cost  $\phi(h, h')$  has to be paid. At the end of the period uncertainty about survival and productivity  $e'$  are revealed. The resulting budget constraint is

$$(1 + r)a + (1 - \phi)h + y(e, j) + tr = a' + h' + p_r h_r + \phi(h, h') + c \quad (11)$$

where in the case of renting  $h' = 0$  and in the case of owning  $h_r = 0$ .

## 2.7. Household problem

The optimization problem of a household is to maximize the expected life-time utility

$$\sum_{j \in \Theta} \beta^{j-1} E_0 \{ S_j u(c_j, h_{o,j}, h_{r,j}) + (1 - S_j) u_b(b_j) \} \quad (12)$$

where  $\beta$  is the discount factor,  $E_0$  is the operator of expectations formulated at the beginning of the first period, and the value of bequest is  $b_j = (1 + r)a_j + (1 - \phi)h_j$ .

Finally, it should be explained that in the above notation the value of the housing stock at the beginning of a period is equal to the value of owned house in the previous period, i.e.  $h_{j+1} = h_{o,j}$ .

The above optimization problem can be written down recursively. At the beginning of a period the household's state  $x$  can be characterized by financial and housing assets, individual productivity

and age  $x = (a, h, e, j)$ . Let  $V_r(x)$  and  $V_o(x)$  denote the value functions of the household in the case of renting and owning, respectively, and  $V(x)$  stand for the proper value function. They can be calculated as the solution of the below optimization problem:

$$\begin{aligned} V_r(x) &= \max_{c, h_r, a'} \{u(c, 0, h_r) + \beta [s_j E(V(x' | x, h' = 0)) + (1 - s_j) u_b(b')]\} \\ V_o(x) &= \max_{c, h_o, a'} \{u(c, h_o, 0) + \beta [s_j E(V(x' | x, h' = h_o)) + (1 - s_j) u_b(b')]\} \\ V(x) &= \max\{V_r(x), V_o(x)\} \end{aligned} \quad (13)$$

subject to the budget constraint (11), the minimum down-payment restriction (10), and conditions:  $c > 0$ ,  $h' \in H_o$ ,  $h_r \in H_r$  and  $b' = (1 + r)a' + (1 - \varphi)h_o$ . The optimal decision is given by the policy functions  $c(x)$ ,  $a'(x)$ ,  $h_o(x)$  and  $h_r(x)$ .

### 3. Equilibrium

Given the heterogeneity of households in terms of the state variable  $x \in X \equiv \mathfrak{R} \times H_o \times E \times \mathfrak{D}$ , to calculate the value of aggregate variables we need some measure of the distribution. Let  $(X, \Xi, \lambda)$ , be a probability space, where  $\Xi \equiv \mathcal{B}(\mathfrak{R}) \times \Pi(H_o) \times \Pi(E) \times \Pi(\mathfrak{D})$ ,<sup>3</sup> and  $\lambda$  is a probability measure, so that for each set  $Y \in \Xi$  the share of individuals with  $x \in Y$  in total population is given by  $\lambda(Y)$ . For a given policy functions  $c(x)$ ,  $a'(x)$ ,  $h_o(x)$  and  $h_r(x)$  the values of aggregate variables consistent with individual behavior can be calculated as:

$$\begin{aligned} \text{Effective labor:} & \quad L = \int z(j, e) d\lambda \\ \text{Consumption:} & \quad C = \int c(x) d\lambda \\ \text{Financial assets:} & \quad A' = \int a'(x) d\lambda \\ \text{Housing assets (owners):} & \quad H_o = \int h_o(x) d\lambda \\ \text{Housing assets (landlords):} & \quad H_r = \int h_r(x) d\lambda \\ \text{Transfers:} & \quad tr = \int (1 - s_j)((1 + r)a'(x) + (1 - \varphi)h_o(x)) d\lambda \\ \text{Transaction costs:} & \quad \Phi = \int \phi(h, h_o(x)) d\lambda \end{aligned} \quad (14)$$

A stationary equilibrium is defined as the policy functions, prices  $w$  and  $r$ , the values of aggregate variables  $K$ ,  $L$ ,  $pen$ ,  $tr$ , the tax rate  $\tau$ , as well as the distribution  $\{\lambda\}$ , that fulfill the following conditions:

1. Given  $w$ ,  $r$ ,  $tr$  and  $pen$  the policy functions are optimal.
2. Aggregate variables are consistent with the policy functions
3. Markets clear:

$$\begin{aligned} A' &= K' + H_r(1 - p_r) + B' \\ H' &= H_o + H_r \\ Y &= C + \delta K' + \varphi H' + \Phi - rB' \end{aligned} \quad (15)$$

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<sup>3</sup> Here  $\mathcal{B}(Y)$  and  $\Pi(Y)$  denote the Borel  $\sigma$ -algebra on  $Y$  and the power set of  $Y$ , respectively.

4. The international capital market condition (1) is met.
5. Factor prices are equal to marginal products as in (3).
6. The budget of the pensions system is balanced as in (7).
7. The distribution of households over the state variable  $x$  is time invariant.<sup>4</sup>

In the stationary equilibrium, the economy is time invariant at the aggregate level. However, at the individual level there is a lot of movement going on. Households are getting older, are hit by idiosyncratic productivity shocks, accumulate financial assets, buy or sell houses, etc. ...

## 4. Parameterization

The model parameters are calibrated partly on the basis of the relevant literature and partly so that the stationary equilibrium matched selected averages for the Polish economy. The benchmark parameter values, which are displayed in Table 1, are as follows.

### 4.1. Demographics

The model period corresponds to one year. Households become economically active at age 20, work for maximum 45 years, and starting from age 65 are on mandatory retirement that lasts up to 21 years. This means that the model describes the behavior of  $J = 66$  cohorts of age from 20 to 85. The conditional survival probabilities  $s_j$  are evaluated on the basis of the data taken from the Demographic Yearbook for Poland (CSO 2010a, p. 309), which were interpolated by a polynomial of degree eight (left panel of Figure 1). The resulting share of retirees in total population amounts to 22.4%, which compares to about 17.3% observed in 2009. It is worthy to notice that this ratio is projected to increase significantly in the forthcoming years.

### 4.2. Individual income process

The deterministic age profile of idiosyncratic productivity  $z_j$  is evaluated with the method proposed by Huggett (1996). We multiply the average wage in different age cohorts, which are taken from the Survey on Structure of Earnings by Occupations (CSO 2009, p. 149), by the labor force participation rates, which is taken from the Labor Force Survey CSO (2011). Subsequently, the series is interpolated by a polynomial of degree four (right panel of Figure 1).

For the stochastic component  $e$ , we follow the literature by assuming that its logarithm is an AR(1) process

$$\ln e' = \rho_e \ln e + \varepsilon, \quad \varepsilon : N(0, \sigma_e^2) \quad (16)$$

Given the lack of estimates for the Polish economy, we set the value of  $\rho_e$  to 0.96 on the basis of estimates for the United States (Floden, Lindé 2001; Storesletten, Telmer, Yaron 2004a; 2004b) and

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<sup>4</sup> A detailed description of this condition can be found e.g. in Chen (2010) or Fernandez-Villaverde and Krueger (2011).



calibrate the value of  $\sigma_e^2$  at 0.025, so that the GINI coefficient of earnings is equal to the observed value of around 0.375. It is worthy to mention that more careful calibration of the labor income process would require an extensive study of the microeconomic data of household budgets. The ongoing study of Urbaniec (2012) is addressing this kind of issues, where the early results indicate that income persistence in Poland is somewhat lower than in the United States.<sup>5</sup>

Using the method of Tauchen (1986), we approximate this continuous AR(1) process by a seven state Markov chain. The resulting states, transition matrix and stationary distribution are:

$$E = \{0.27, 0.40, 0.58, 0.84, 1.23, 1.79, 2.61\}$$

$$\pi = \begin{bmatrix} 0.817 & 0.182 & 0.001 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.084 & 0.758 & 0.158 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.099 & 0.764 & 0.137 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.117 & 0.766 & 0.117 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.137 & 0.764 & 0.099 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.158 & 0.758 & 0.084 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.001 & 0.182 & 0.817 \end{bmatrix} \quad (17)$$

$$\bar{\pi} = [0.058 \quad 0.125 \quad 0.200 \quad 0.234 \quad 0.200 \quad 0.125 \quad 0.058]'$$

As for the productivity at the initial age, following Hugget (1996) and taking the evidence that earnings inequality is increasing with age (Heathcote, Storesletten, Violante 2005), we set its variance at two thirds of the unconditional variance of  $e$ .

Subsequently, we set  $\chi$  to 0.4 so that the value of pension equal to 40% of average pre-tax labor income, which broadly corresponds to the data from the OECD (2009) report. The implied tax rate is 11.5%. It should be emphasized that in the model the value of pension does not depend on earnings history. This assumption, which does not fully reflects the functioning of the pension system in Poland, was made due to computational reasons.

Finally, to express the value of income in terms of Polish zlotys (its 2009 value), we assume that the average before tax wage income of households  $\bar{w}$  is 67 000 PLN and the pension income of households is 27 000 PLN.<sup>6</sup> Given that we do not model the public sector, this means that the average after tax wage is around 60 000 PLN, which broadly corresponds to the average annual income of employees (CSO 2010b, p. 55).

<sup>5</sup> This is taken into account by calibrating the value of  $\rho_e$  somewhat lower than the median of estimates for the United States.

<sup>6</sup> The above numbers refer to the income of a household, which on average consists of 3.3 and 2.0 persons for employees and retirees, respectively.

### 4.3. Preferences

The discount factor  $\beta$  is fixed at 0.98 and the relative risk aversion coefficient  $\eta$  is chosen to be 1.5, which is in the middle of the range commonly used in the literature and consistent with the estimates of Gourinchas and Parker (2002). For the remaining parameters of the utility function (4), we follow Kiyotaki, Michaelides, Nikolov (2007) and fix their values at  $\theta = 0.75$  and  $\vartheta = 0.9$ . The chosen share of non-housing consumption in the utility function  $\theta$  broadly reflects the estimates of Li, Liu, Yao (2009) and Bajari et al. (2010) for the U.S. economy, at the same time being consistent with the Polish data on household spending (CSO 2010b, p. 86). For the parameter describing the fraction of utility loss from renting  $\vartheta$ , under the benchmark parameterization its value is consistent with the steady-state homeownership ratio of 87.5%. This ratio might seem high in comparison to the data of the European Union Statistics on Income and Living Conditions (EU-SILC), which shows that it stood at 69% in 2009. We explain this discrepancy by the significant share of public housing, which is not included in the model: according to the EU-SILC data, 29% of households were living in this type of apartments.

As regards the bequest motive, we calibrate the altruism parameter  $\kappa = 10$  so that the marginal propensity to consume in the last period was about 0.25 (Cagetti 2003). This means that a household who is sure to die would leave about 75% of her or his wealth to heirs. It should be noted that in the model the bequest motive helps to prevent households from selling their houses in the last periods of their life.

### 4.4. Technology and the housing market

We model an open economy where households have access to foreign capital subject to international markets frictions. We assume  $r^* = 0.02$  and  $\xi = 0.01$  so that an increase in the foreign debt by 10% of GDP would lead to an increase in the level of domestic interest rate by 0.1 percentage point. The capital share  $\alpha$  and the depreciation rate  $\delta$  are set to standard values of 0.3 and 0.10, respectively. For the maintenance costs of housing, we fix  $\varphi$  at 0.03, which is close to the estimate of Li, Liu, Yao (2009) for the U.S. economy, and broadly consistent with values usually assumed in the literature (Yao, Zhang 2005; Li, Yao 2007; Iacoviello, Pavan 2009). The example interpretation is as follows: one needs to spend 750 PLN per month to keep the value of a 300 000 PLN house unchanged, which is close to the data in NBP (NBP 2010, Fig. 10). As regards the transaction costs of selling/buying a house, we take the values of  $\phi_1 = 0.02$  and  $\phi_2 = 0.03$ . We assume that the transaction costs of selling include the intermediation fee only, whereas the transaction costs of buying also cover the tax on civil law transactions and the cost of notarial acts.<sup>7</sup> It can be noticed that our choice is somewhat lower than the usual values for the U.S. economy (Li, Yao 2007; Yang 2009; Bajari et al. 2010), and does not consider any non-financial costs of moving such as time devoted to find a new house. The last parameter we calibrated describes the minimum down-payment requirement  $\gamma$ . In the benchmark model we take  $\gamma = 0.1$ , which means that the own contribution of households buying a house cannot be lower than 10% of the property value. We justify our choice by the data from

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<sup>7</sup> The usual intermediation fee in Poland is 3%, but not all households are participating in the market through real estate agencies. Moreover, in many cases the fee is reduced. As regards the tax on civil law transactions, its rate is 2% for transactions on the secondary market and null for transactions on the primary market.

the AMRON-SARFiN report (2011), which shows that the fraction of newly granted mortgages with the loan-to-value ratio exceeding 80% in total mortgages fluctuates in the range from 25% to 55%. This means that setting  $\gamma = 0.2$  would be too conservative.

Finally, we assume that the minimum-size house available for purchase  $h_1$  costs triple the average annual pre-tax household income, i.e. 200 000 PLN. Given that the average price of a square meter stood at about 4 000 PLN in 2009, this corresponds to a typical 2-room apartment of 50 square meters.<sup>8</sup> The other house values available for purchase are 300 000 PLN, 400 000 PLN and 600 000 PLN. The size of rented apartments can also take the value  $h_0$  equal to 100 000 PLN. The above assumptions, together with the down-payment restriction, prevents asset-poor people from becoming homeowners too quickly.

## 5. The results

The model has no closed-form solution, and therefore we have to solve it numerically. For that purpose we discretize the space for financial assets  $a$  over grid points  $A = \{a_1, a_2, \dots, a_L\}$ , where we set the bounds  $a_1$  and  $a_L$  at levels not constituting a constraint for the optimization problem. Moreover, we do not restrict households to select  $a'$  lying on the grid  $A$ , but instead use the golden section search method to cover any intermediate choices. Subsequently, we apply the following algorithm to calculate the stationary equilibrium:

1. Set the value of  $r$  and  $tr$ .
2. Compute  $w$  and  $K$  consistent with  $r$  with (3).
3. Solve the optimization problem (13) by backward induction and compute the policy functions for each  $x \in X$ .
4. Compute the distribution  $\lambda$  by forward induction.
5. Calculate the value of aggregate variables with (14).
6. Calculate the value of net foreign assets with (15).
7. Calculate the value of the real interest rate with (1).
8. Check whether the values of  $r$  and  $tr$  calculated in steps 5 and 7 are equal to those from step 1. If yes, stop. Otherwise go to step 1 and update  $r$  and  $tr$ .

### 5.1. Life cycle profile

Figure 2 presents the life-cycle paths of key model variables for the benchmark parameterization. We show the average values for all age cohorts as well as the individual values for three randomly selected households, assuming that they survive till the last period. The left-upper panel presents the after-tax annual labor income. The average value is hump-shaped, since it is determined by the age profile of individual productivity (right panel of Figure 1). Individual income, conditional on age, can take seven different values, depending on the realization of the idiosyncratic productivity  $e \in E$ . It can be noticed that the first household was the most fortunate in terms of labor income, whereas the third one was relatively unlucky.

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<sup>8</sup> See the CSO publication: [http://www.stat.gov.pl/gus/5840\\_4671\\_ENG\\_HTML.htm](http://www.stat.gov.pl/gus/5840_4671_ENG_HTML.htm).

The exogenous differences in life-time earnings are reflected in the decisions taken by households related to consumption, savings and housing. The right-upper panel of Figure 1 shows that the average lifetime profile of consumption is also hump-shaped, but its variability is much lower than that of income. In the initial years households are consuming relatively little due to three reasons: they earn relatively little, the borrowing constraint (10) tends to bind, and they accumulate assets for the down-payment necessary to buy their first house. This chart also shows that individual consumption profile reacts to changes in individual income, which in our model is relatively persistent. This is in line with the empirical evidence for the U.S. economy, as presented by e.g. by Carroll and Summers (1989). The left-bottom panel presents the value of financial assets. In initial periods households tend to take loans for house purchases. At around the age of forty, mortgages are repaid and financial savings are accumulated to self-insure against expected income decrease in the retirement period. In the last periods households are using their life-time savings to keep consumption above their income, which is determined by the value of pension. Finally, the right-bottom panel of Figure 1 presents the value of owned houses. The average house size increases till around age 50 and then flattens out. At the individual level, households change the size of owned house very infrequently: once or twice during the lifetime. The relatively well-earning household 1 decided to buy the first apartment at age 26 and to move up on the property ladder at age 33 and 42. On the other hand, household 3 bought a 50 square meter apartment at age of 27 and lived there for the entire lifespan.

## 5.2. Comparison of the benchmark and alternative economy

The model allows us to examine the impact of changes to the down-payment requirement on the economy. Towards that aim we compare the stationary equilibria in economies with various values of the parameter  $\gamma$ . In the benchmark economy  $\gamma = 0.1$ , whereas in the alternative ones it is set to 0.3, 0.5 and 0.7, respectively.

Table 2 shows that a change in the minimum down-payment requirement has a negligible effect on the level of the real interest rate, the capital-output ratio or the share of investment and consumption in output. However, it has a significant impact on the aggregate level of debt, the homeownership ratio, the average age of first house purchase or the composition of the housing stock. An increase of the minimum down-payment requirement from 10% to 70% leads to an eightfold decrease in household debt from 43.0% of GDP to 5.2% of GDP, a decline in the homeownership rate from 87.6% to 68.6% and a rise in the average age of first house purchase from 26.8 to 37.6. This is reflected in a positive relationship between the minimum down-payment requirement and the share of the housing stock held by landlords.

Figure 2 illustrates the effects of changes in the down-payment requirement on different age cohorts. It shows that both: the average life-time paths of non-housing consumption and used house size are little affected by changes in  $\gamma$ . In contrast, the impact on the ownership status of occupied property is significant. For low values of  $\gamma$  almost all households over thirty live in owned houses, whereas for high values of  $\gamma$  a large fraction of households above thirty is still renting houses. This difference is reflected in the life-time path of financial assets and debt.

The above considerations show that, given well functioning rental market, changes in the minimum down-payment requirement do not affect significantly the size of occupied house or the level of consumption in the life-cycle. At the same time, they show that changes in the down-payment requirements have a significant impact on the ownership structure of houses. This affects welfare, because households prefer owning a house to renting it.

### 5.3. Welfare analysis

We continue by evaluating the welfare effects of changes in the minimum down-payment restriction. We do this by comparing the ex-ante expected life-time utility of the newborn cohort in the stationary equilibria of the benchmark and alternative economies. It should be noted that since we focus on the stationary equilibria, we investigate only the long-term effects of changes in the down-payment requirement, abstracting from any costs or benefits related to the transition dynamics. Moreover, our analysis does not consider any welfare effects of changes in the loan-to-value ratio related to the stability of the financial system.

Our first question is how much would a newborn household with the individual productivity  $e$  be eager to pay in terms of PLN to live in the benchmark economy, in which  $\gamma = 0.1$ , rather than in the alternative one with  $\gamma = \gamma^*$ . In the benchmark economy, the ex-ante expected life-time utility of such a household is equal to its value function  $V(0, 0, e, 1)$  defined in (13). Let  $V^*$  denote the value function of the household in the alternative economy. To answer the posed question we need to solve for  $v(e)$  in the below equation:

$$V(0, 0, e, 1) = V^*(v(e), 0, e, 1) \quad (18)$$

The value of  $v(e)$  can be interpreted as the value of financial assets that has to be given to the household to compensate for the utility loss related to higher minimum down-payment requirement.

The third column of Table 3 presents reports the welfare effects of increasing  $\gamma$  from 10% to 30%, 50% and 70%, respectively. Depending on the individual productivity  $e$ , a newborn household would be eager to pay between 218 PLN and 463 PLN to live in the benchmark economy in comparison to the alternative one with  $\gamma^* = 0.3$ . An increase in the minimum down-payment requirement from 10% to 70% would require a compensation ranging between 346 PLN and 1145 PLN.

The second question we ask is how much do we need to increase the consumption index (as defined in equation 4) and the bequest value of a newborn household with individual productivity  $e$  during entire lifespan so that she or he would be indifferent between living in the benchmark and alternative economy. Let us notice that given (12) the value function of a newborn household is

$$\begin{aligned} V &= \sum_{j \in \mathfrak{H}} \beta^{j-1} E_0 \{ S_j u(c_j, h_{o,j}, h_{r,j}) + (1 - S_j) u_b(b_j) \} \\ V^* &= \sum_{j \in \mathfrak{H}} \beta^{j-1} E_0 \{ S_j u(c_j^*, h_{o,j}^*, h_{r,j}^*) + (1 - S_j) u_b(b_j^*) \} \end{aligned} \quad (19)$$

where star (\*) denotes the alternative economy as in (18). The answer to the question is the value of  $\omega$  that solves

$$V = \sum_{j \in \mathfrak{D}} \beta^{j-1} E_0 \{ S_j u((1+\omega)c_j^*, (1+\omega)h_{o,j}^*, (1+\omega)h_{r,j}^*) + (1-S_j) u_b((1+\omega)b_j^*) \} \quad (20)$$

Given the form of the utility and bequest functions (equations 4 and 5), the above problem can be expressed as

$$V = (1+\omega)^{1-\eta} V^* \quad (21)$$

which means that

$$\omega(e) = \left( \frac{V(0, 0, e, 1)}{V^*(0, 0, e, 1)} \right)^{\frac{1}{1-\eta}} - 1 \quad (22)$$

The results are reported in the forth column of Table 3. It shows that to compensate newborn households for an increase in the minimum down-payment requirement from 10% to 30%, the lifetime consumption index has to be augmented in the range between 0.07% and 0.15%, depending on the individual productivity  $e$ . Moreover, an increase in the minimum down-payment requirement from 10% to 70% requires a compensation ranging between 0.17% and 0.30%.

The last question we pose is about the aggregate welfare loss related to an increase in the minimum down-payment restriction. We consider the utilitarian welfare function:

$$W = \sum_{e \in E} V(0, 0, e, 1) \pi_0(e) \quad (23)$$

which sums the utility of newborn households. Here  $\pi_0(e)$  is the share of a household with initial productivity  $e$  in total population of the newborn cohort ( $\pi_0(e) = \lambda(0, 0, e, 1)/\mu_1$ ). The aggregate welfare loss is equal to:

$$\bar{\omega} = \left( \frac{W}{W^*} \right)^{\frac{1}{1-\eta}} - 1. \quad (24)$$

where  $W^*$  is the welfare function in the alternative economy.

The last row of Table 3 indicates that the aggregate welfare loss due to an increase in the minimum down-payment requirement from 10% to 30%, 50% and 70%, amounts to 0.12%, 0.23% and 0.25% of life-time consumption index, respectively.

## 6. Conclusions

In this paper we have analyzed the welfare effects of changes in the minimum down-payment requirement. For that purpose we have developed a life-cycle model with individual income

uncertainty, in which households derive utility from consumption of non-housing goods, housing services and bequests. In the model economy houses can be owned or rented, where owning a house is preferred to renting it. For that reason households want to buy a house as soon as possible. However, they are constrained to do so due to the existence of the minimum down-payment requirement.

We have showed that the effects of changes in the minimum down-payment requirement on the macroeconomic aggregates such as the capital stock, consumption or investment are negligible. On the other hand, we have found that these changes have a tremendous impact on the level of household debt, the ownership ratio or the time of first house purchase. Taking into account that in our model households prefer owning a house to renting it, tougher credit conditions lead to a welfare loss. Our calculations show that in the long run these costs are equal to from 0.07% to 0.30% of the life-time consumption index.

It is important to note that we have investigated only one aspect of the above issue, namely how changes in the minimum down-payment requirement affect welfare in the long-run through the effects on the homeownership ratio. There are several natural extensions to our analysis. First, it would be interesting to investigate the welfare effects of changes in the minimum down-payment requirements taking into account transitional dynamics, as in the model of Chambers, Garriga and Schlagenhaut (2009). One could expect that tougher financing conditions could temporarily dampen house prices, which would be beneficial for house buyers and detrimental for house owners. Second, we have assumed a perfectly competitive rental market so that, neglecting the transaction costs, the prices of owning and renting a house are the same. In practice the maintenance costs of rented houses tend to be higher due the moral hazard problem (Henderson, Ioannides 1983). Moreover, the underdevelopment of the private rental market in Poland (NBP 2010) can additionally increase the price of renting. This means that an increase in the minimum down-payment requirement, which leads to a decrease in the homeownership ratio, is welfare detrimental not only due to the preference of owning but also because renting is more expensive. Finally, since in our model there is no default or aggregate risk, it is not well suited to analyze the effects of changes in the minimum down-payment restrictions on the stability of the financial system. We leave all these topics for further research.

## References

- Aiyagari S.R. (1994), Uninsured idiosyncratic risk and aggregate saving, *The Quarterly Journal of Economics*, 109(3), 659–684.
- AMRON (2011), *Ogólnopolski raport o kredytach mieszkaniowych 2/2011*, <http://www.amron.pl>.
- Bajari P., Chan P., Krueger D., Miller D. (2010), *A dynamic model of housing demand: Estimation and policy implications*, Working Papers, 15955, National Bureau of Economic Research.
- Ca' Zorzi M., Rubaszek M. (2012), On the empirical evidence of the intertemporal current account model for the euro area countries, *Review of Development Economics*, 16(1), 95–106.
- Cagetti M. (2003), Wealth accumulation over the life cycle and precautionary savings, *Journal of Business & Economic Statistics*, 21(3), 339–353.
- Carroll C., Summers L.H. (1989), *Consumption growth parallels income growth: Some new evidence*,

- Working Papers, 3090, National Bureau of Economic Research.
- Carroll C.D. (1997), Buffer-stock saving and the life cycle/permanent income hypothesis, *The Quarterly Journal of Economics*, 112(1), 1–55.
- Chambers M., Garriga C., Schlagenhaut D.E. (2009), Accounting for changes in the homeownership rate, *International Economic Review*, 50(3), 677–726.
- Chen K. (2010), A life-cycle analysis of social security with housing, *Review of Economic Dynamics*, 13(3), 597–615.
- CSO (2009), *Structure of Wages and Salaries by Occupations in October 2008*, Central Statistical Office, Warsaw.
- CSO (2010a), *Demographic Yearbook of Poland*, Central Statistical Office, Warsaw.
- CSO (2010b), *Household Budget Surveys for 2009*, Central Statistical Office, Warsaw.
- CSO (2011), *Labor Force Survey in Poland, IV Quarter 2010*, Central Statistical Office, Warsaw.
- Deaton A. (1991), Saving and liquidity constraints, *Econometrica*, 59(5), 1221–1248.
- Fernandez-Villaverde J., Krueger D. (2011), Consumption and saving over the life cycle: How important are consumer durables?, *Macroeconomic Dynamics*, 15(5), 725–770.
- Floden M., Lindé J. (2001), Idiosyncratic risk in the United States and Sweden: Is there a role for government insurance?, *Review of Economic Dynamics*, 4(2), 406–437.
- Gourinchas P.-O., Parker J.A. (2002), Consumption over the life cycle, *Econometrica*, 70(1), 47–89.
- Heathcote J., Storesletten K., Violante G.L. (2005), Two views of inequality over the life cycle, *Journal of the European Economic Association*, 3(2–3), 765–775.
- Henderson J.V., Ioannides Y.M. (1983), A model of housing tenure choice, *American Economic Review*, 73(1), 98–113.
- Huggett M. (1993), The risk-free rate in heterogeneous-agent incomplete-insurance economies, *Journal of Economic Dynamics and Control*, 17(5–6), 953–969.
- Huggett M. (1996), Wealth distribution in life-cycle economies, *Journal of Monetary Economics*, 38(3), 469–494.
- Iacoviello M., Pavan M. (2009), *Housing and debt over the life cycle and over the business cycle*, Working Papers, 09-12, Federal Reserve Bank of Boston.
- Kiyotaki N., Michaelides A., Nikolov K. (2007). *Winners and losers in housing markets*, Conference Paper Series, 0705, Centre for Dynamic Macroeconomic Analysis.
- Li W., Liu H., Yao R. (2009), *Housing over time and over the life cycle: a structural estimation*, Working Papers, 09-7, Federal Reserve Bank of Philadelphia.
- Li W., Yao R. (2007), The life-cycle effects of house price changes, *Journal of Money, Credit and Banking*, 39(6), 1375–1409.
- NBP (2010), *Report on the Polish residential market in 2002–2009*, National Bank of Poland, Warsaw.
- OECD (2009), *Pensions at a glance 2009: retirement-income systems in OECD countries*, OECD, Paris.
- Ortalo-Magne F., Rady S. (2006), Housing market dynamics: on the contribution of income shocks and credit constraints, *Review of Economic Studies*, 73(2), 459–485.
- Rabanal P., Crowe C.W., Igan D., Dell’Ariccia G. (2011), *How to deal with real estate booms: lessons from country experiences*, Working Papers, 11/91, International Monetary Fund.
- Storesletten K., Telmer C.I., Yaron A. (2004a), Consumption and risk sharing over the life cycle, *Journal of Monetary Economics*, 51(3), 609–633.
- Storesletten K., Telmer C.I., Yaron A. (2004b), Cyclical dynamics in idiosyncratic labor market risk, *Journal of Political Economy*, 112(3), 695–717.



- Tauchen G. (1986), Finite state Markov-chain approximations to univariate and vector autoregressions, *Economics Letters*, 20(2), 177–181.
- Urbaniec A. (2012), *Life cycle income and consumption patterns in transition*, unpublished report.
- Yang F. (2009), Consumption over the life cycle: How different is housing?, *Review of Economic Dynamics*, 12(3), 423–443.
- Yao R., Zhang H.H. (2005), Optimal consumption and portfolio choices with risky housing and borrowing constraints, *Review of Financial Studies*, 18(1), 197–239.
- Zeldes S.P. (1989), Optimal consumption with stochastic income: deviations from certainty equivalence, *The Quarterly Journal of Economics*, 104(2), 275–98.

## Appendix

Table 1  
Benchmark parameterization

<b>Demographics</b>		
Number of cohorts	$J$	66
Number of working cohorts	$\tilde{J}$	45
Surviving probabilities	$s_j$	CSO (2010a)
<b>Individual income process</b>		
Age profile of productivity	$z_j$	CSO (2009; 2011)
Persistence of idiosyncratic productivity	$\rho$	0.96
Variance of idiosyncratic productivity	$\sigma_e^2$	0.025
Pension replacement rate	$\chi$	0.4
<b>Preferences</b>		
Discount factor	$\beta$	0.98
Risk aversion	$\eta$	1.50
Share of non-housing consumption in utility	$\theta$	0.75
Utility loss from renting	$\vartheta$	0.90
Bequest preference	$\kappa$	10
<b>Technology and housing market</b>		
Openness of the economy	$\xi$	0.01
Capital share in production function	$\alpha$	0.30
Depreciation rate	$\delta$	0.10
Housing maintenance cost	$\varphi$	0.03
Transaction cost of selling	$\phi_1$	0.02
Transaction cost of buying	$\phi_2$	0.03
Minimum down-payment ratio	$\gamma$	0.10

Table 2  
Comparison of benchmark and alternative economies

	$\gamma = 0.1$	$\gamma = 0.3$	$\gamma = 0.5$	$\gamma = 0.7$
Interest rate in %	2.369	2.374	2.388	2.382
Capital to GDP ratio	2.426	2.425	2.422	2.423
Investment as % of GDP	20.5	20.5	20.4	20.4
Housing investment as % of GDP	11.4	11.4	11.5	11.5
Consumption as % of GDP	63.0	63.0	62.9	62.9
Fraction of households with debt	29.2	21.3	15.3	10.7
Household debt as % of GDP	43.0	23.2	11.8	5.2
Homeownership ratio	87.6	79.6	73.2	68.6
Av. age of first house purchase	26.8	31.7	34.9	37.6
Share of rented houses in total housing stock	7.0	13.5	19.1	23.1

Table 3  
Welfare loss due to increase in down-payment requirements

$e$	Share $\pi_0(e)$	PLN $-v(e)$			% of lifespan consumption $-\omega(e)$		
		$\gamma^* = 0.3$	$\gamma^* = 0.5$	$\gamma^* = 0.7$	$\gamma^* = 0.3$	$\gamma^* = 0.5$	$\gamma^* = 0.7$
0.27	2.1	218	421	449	0.132	0.252	0.268
0.40	9.0	258	344	346	0.142	0.263	0.285
0.58	23.1	247	468	508	0.146	0.272	0.293
0.84	31.6	281	541	597	0.127	0.240	0.263
1.23	23.1	290	584	657	0.096	0.191	0.214
1.79	9.0	307	650	755	0.073	0.152	0.177
2.61	2.1	463	956	1145	0.075	0.153	0.183
Aggregate					0.120	0.229	0.251

Notes: welfare loss in comparison to the benchmark economy with  $\gamma = 0.1$ .

Figure 1  
Survival probability and median productivity

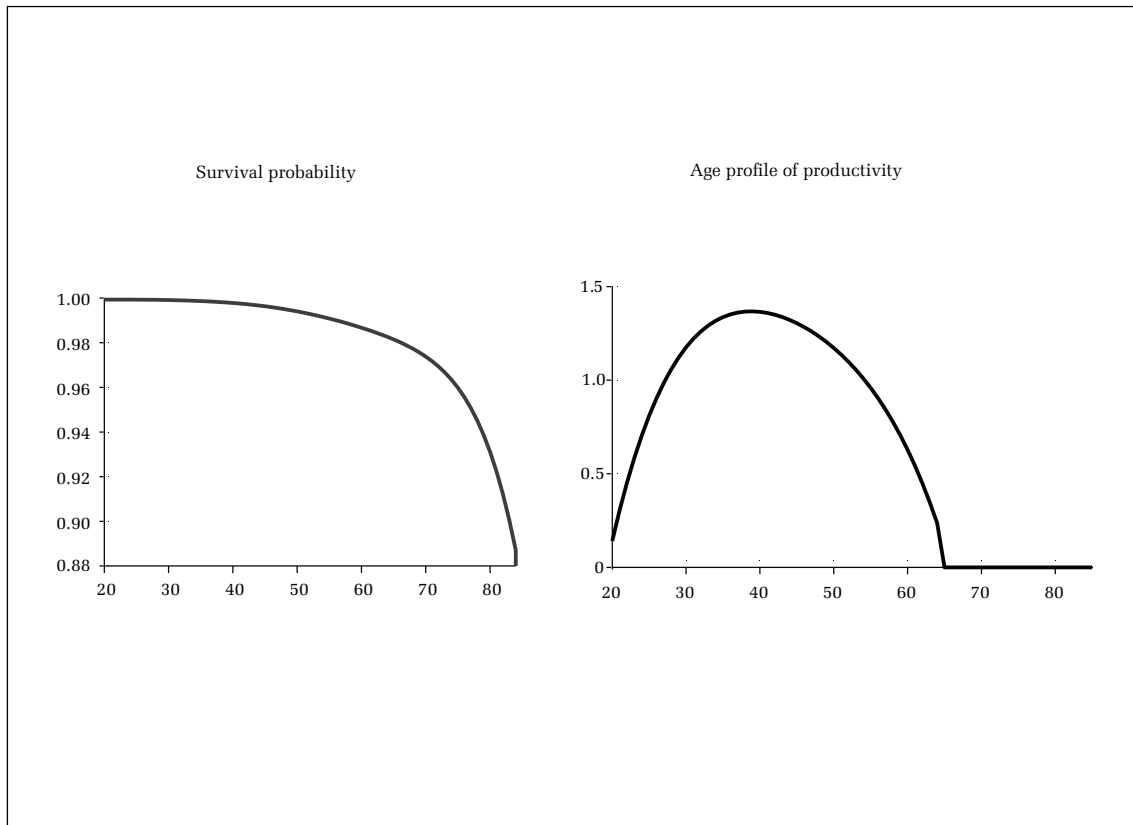
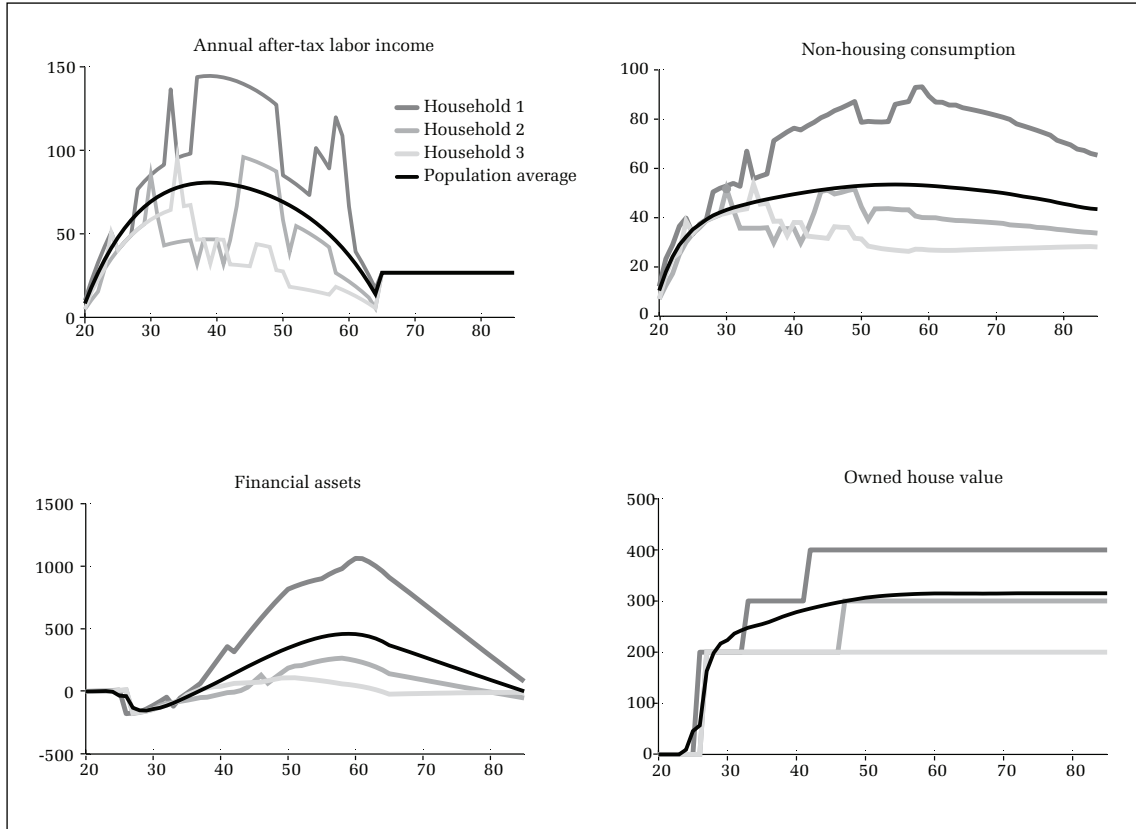
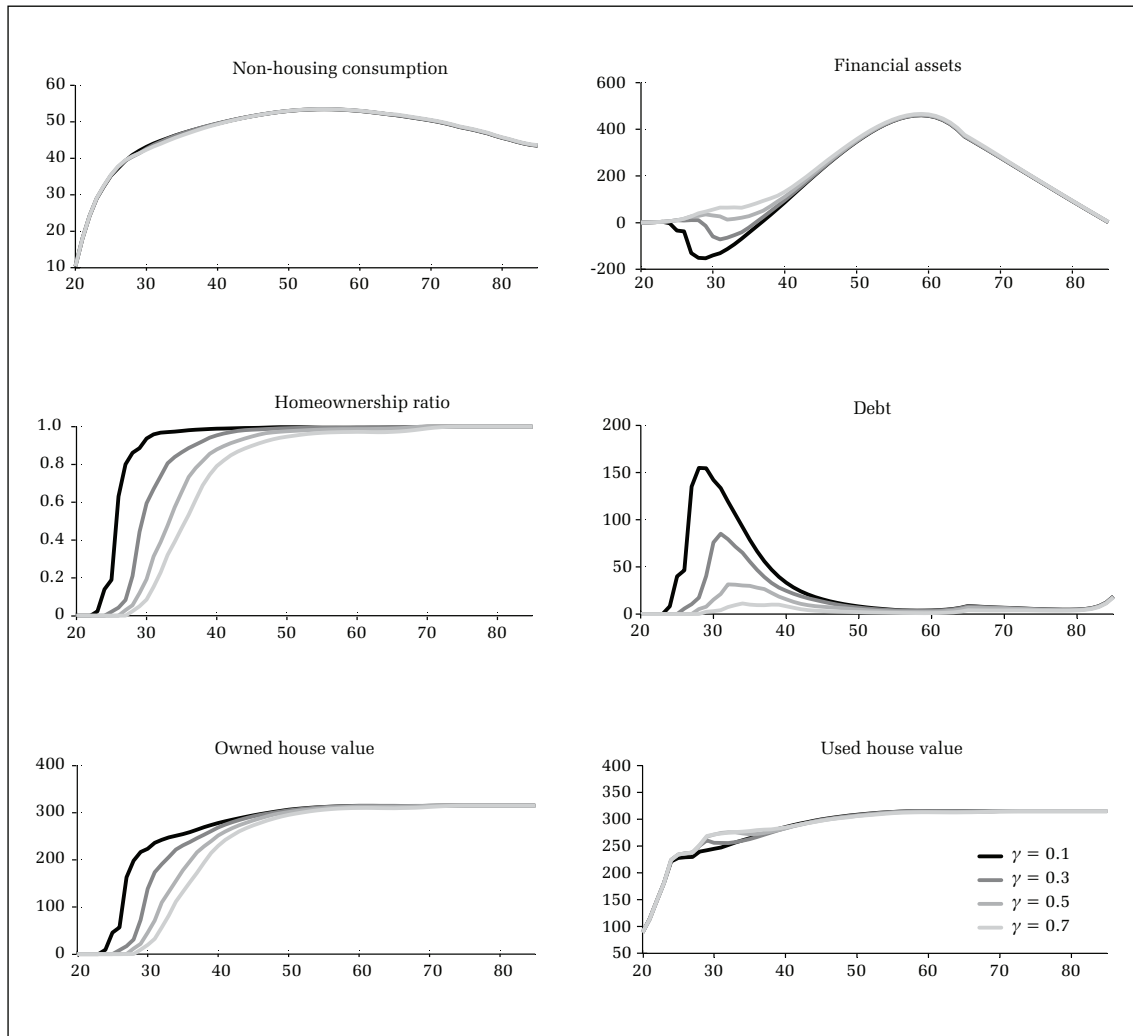


Figure 2  
Life cycle path



Notes: the figure presents the average values for all age cohorts as well as the individual values for three randomly selected households, assuming that they survive till the last period. All values are expressed in th. PLN.

Figure 3  
Comparison of benchmark and alternative economies



Notes: average values for each age cohort. All values, but the homeownership ratio, are expressed in th. PLN.

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