

Demand functions in Polish Treasury auctions

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Abstract

I introduce a new approach to modeling aggregate bidding functions (demand functions) submitted by participants of share auctions, the one based on (scaled) normal cumulative distribution functions. I provide a simple model illustrating how normal cdf-shaped demand might arise. Then, using new data from the Polish Treasury securities auctions, I show first, that assumptions of the model underlying the normal cdf specification fit the stylized characteristics of the data set and, second, that this approach actually generates a slightly better fit than the traditional approximation by logistic function. I also relate the parameters of the fitted function to economic variables known prior to the auction. This method appears to be a useful tool for early detection of slumps in the performance of a particular auction design.

Keywords: Treasury auctions, normal cumulative distribution function, underpricing

JEL: C31, C51, D44

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1. Introduction

Despite the spectacular progress in our knowledge on divisible goods auctions in the last years (see e.g. Daripa 2001), it appears that researchers have yet to deliver clear and uncontested answers to some of the questions that appear crucial from the policy-maker's point of view. The most important examples are the four-decades-long discussions regarding the relative performance of different pricing rules and magnitude of underpricing in Treasury auctions when compared to the secondary market. Even though these inter-related issues constituted (for a good reason) the essence of research on Treasury auctions, the answers are not clear-cut; theoretical results seem to be quite sensitive to particular assumptions. It seems especially difficult to derive a general form of equilibrium strategies in discriminatory share auctions; see however Wang, Zender (2002) as well as Back, Zender (1993), Hortaçsu (2002a) and Viswanathan, Wang, Witelski (2000) for special cases.

One general conclusion from this literature seems to be that in such a complex environment there typically arise multiple equilibria. It seems then that the most promising way to get insight into the impact of possible changes in the auction design, economic situation etc. is to analyze the actual bidding behaviour. In this way the burden of investigation is transferred to the empiricists.

Unfortunately, the quest to measure and compare underpricing across different auction types also faced substantial difficulties. Studies comparing different countries are of limited value, due to several institutional differences that may be confused with the pricing rule effect. Within-country comparisons are only possible in the rare cases when the Treasury decided to change the auction format. Even then, it cannot be taken for granted that relevant economic variables did not change over the period. Furthermore, it can be argued that it takes time for the bidders to adjust to the new system, thus the period directly following the introduction of the new rule may be atypical. Worse still, change of the pricing rule is likely to be endogenous – if, for example, its performance is subject to random variation and the change of the pricing rule occurs after a longer period of dissatisfying results, the new system will probably outperform the old one due to the regression to the mean rather than any substantial advantage (just as rain summoning works quite fine if you wait long enough and duration of droughts exhibits increasing hazard rate).

Given these obstacles, it should not be surprising that results are not univocal. To mention just a few examples, Umlauf (1993) found underpricing of 0.018% of the face value in the case of discriminatory auctions of Mexican Treasury securities and no significant underpricing in uniform price auctions. Nyborg and Sundaresan (1996) and Goldreich (2003) also reported higher revenue for uniform pricing rule, introduction of which lowered underpricing by some 0.2 basis points in the USA, constituting a significant improvement. Using a different technique, Heller and Lengwiler (1997) find qualitatively similar results for Swiss government auctions.

On the other hand, Bjonnes (2001; 2002) found opposite evidence in Norway. Also Hortaçsu (2002b), using a clever counterfactual analysis based on bootstrap estimation of unobservable true valuations, concluded that in Turkey discriminatory auctions yield higher revenues than uniform auctions. Fevrier et al. (2002) came to a similar conclusion using French data. Hamao and Jagadeesh (1998) found no significant underpricing in discriminatory auctions in Japan.

In view of the above-mentioned difficulties and generally mixed results, a new approach to investigation into Treasury auctions has emerged in the last ten years, which, rather than aggregate statistics, analyzes actual bid functions submitted by the buyers. In this way the researchers seek to identify determinants of actual bidding behaviour and resulting proceeds from the auction. The main research route is to model the, typically S-shaped, bid functions as logistic functions. This functional form is attractive due to its flexibility and the fact that it might be obtained as an integral of a bell-shaped distribution of yield rates in individual bids. The variations of the estimated parameters of the logistic function over time can be assumed to be random (Boukai, Landsberger 1999; Berg et al. 1999), or to depend on other economic variables (Preget, Waelbroeck 2005; Özcan 2004). The latter approach seems more promising as it allows generating out-of-sample predictions of bid functions, and thus cut-off price conditional on changes in explanatory variables (Preget, Waelbroeck 2005). As shown by Özcan (2004), the logistic function approach can help us to compare the performance of different pricing rules. His strategy is to estimate the relationships between certain economic variables and (parameters of) bid functions under the uniform and discriminatory pricing rules separately (which is possible thanks to switching from one mechanism to the other which occurred in his sample of Turkish Treasury auctions) and simulate the hypothetical bids that would have been submitted under the counter-factual pricing rule. He concluded that the discriminatory pricing rule would have outperformed the uniform rule. Preget and Waelbroeck (2005), who only have data on discriminatory auctions, investigated potential results of hypothetical design changes within this pricing rule. They found, *inter alia*, that the Treasury should avoid running too many auctions on the same day and that reopening of a particular line of bills generates additional costs, compared to launching a new issue. Vargas (2003) used estimation of bidding functions in uniform price Treasury auctions in Argentina to compute the (revenue-relevant) level of risk-aversion prevalent among the bidders.

This paper continues this line of research, yet, introducing some substantial changes in the methodology. First, I approximate the bids using normal cumulative distribution function (cdf) rather than logistic function. While logistic approach might be justified on the grounds that the two functions differ only slightly and logistic function is somewhat more handy from the computational viewpoint, I argue that normal cdf is more appropriate as aggregation of individual demand functions. To illustrate the point, I sketch a model of dealer-specific bid functions that is consistent with data features and lends support to the normal cdf specification of aggregate demand functions. I also show that normal cdf model performs at least as well as logistic function: it generates a better fit in majority of auctions and slightly lower overall sum of squared residuals. Finally, I am able to contribute to the discussion on the performance of particular Treasury auction mechanisms and, consequently, rents obtained by the primary dealers, by predicting parameters of the fitted demand functions basing on the information available prior to the auction. Any substantial deviations from the forecast values might indicate a systemic change in the behaviour of primary dealers. This can, for example, result from the emergence of a collusive agreement. Likewise, behavioural results of institutional modifications (i.e. changes in the auction design) can be assessed in a handy way in terms of corresponding shifts in demand parameters. In the case of Poland, the implementation of new regulations for supplementary fixed-price tenders in 2005 calls for such analysis and will be addressed elsewhere.

The second contribution is that this is one of the very few papers on Treasury auctions in a former communist country (and the only one that I know of that models individual demand functions). Given that features of the auction design, secondary market thickness, market power and links between primary dealers etc. differ significantly among countries and may substantially affect auction results, an analysis of data from economies with varying background is highly desirable. Here, I indeed find some unusual features of the data, most notably underpricing being much higher than in most previous studies.¹

The remaining part of the paper is organized as follows. Section 2 describes the Polish Treasury auctions data, including evidence of substantial underpricing relative to the secondary market. Section 3 explains the methodology of fitting logistic and normal cdf curves to the aggregated demand functions and estimating their parameters. This section also discusses the model of individual bidder behaviour supporting the normal cdf curve approach. Section 4 presents the results of the estimation procedures and Section 5 displays the relationship between the parameters of the fitted functions and underlying economic variables. Conclusion is presented in Section 6.

2. Description of data

The paper makes use of two data sets: the primary market data set reporting individual bids in two-year bond auctions and fifty-two week bill auctions and the secondary market data set containing yields of securities of the same duration. In this section I give a brief description of both.

Fifty-two week Treasury bills are the most important short-term government security in Poland. The Ministry of Finance (MF), represented by the Central Bank, auctions approximately USD 250 million of those every Monday. Tenders of 2-year zero-coupon bonds are organized on a monthly basis with the face value of approximately USD 750 million at every auction. Bids are required to be submitted before 11.00 am of the specified day (Wednesday or Thursday) and results of the auctions are published within an hour. Since the beginning of 2003, only primary dealers have had the right (and obligation) to submit sealed bids at auctions and resell securities on the secondary market. There were 12 such dealers in the analyzed period.² Bids are formulated in terms of price per PLN 10 000 (52 week bills) or PLN 1000 (2Y bonds) of the face value. No deposits against the submitted bids are required. Payments follow within two days after the auction in the case of Treasury bills and up to two weeks in the case of 2Y bonds. The minimum bid is PLN 1 million (approximately USD 280 thousand) and the number of bids is unlimited. The MF uses the discriminating (multi-price) rule and noncompetitive bids are not allowed. In general, the supply is known in advance. It is at the MF's discretion to reduce the amount sold in the case of dissatisfying demand, but this occurs on very rare occasions. In the case of 2Y bonds the MF may however, and frequently does, offer additional bonds on the next day, at fixed price equal to the weighted average of the accepted bids.

¹ This finding may partly be due to imperfect secondary data source, see next section.

² Additionally, Bank Gospodarstwa Krajowego, while not a dealer, was allowed to participate in auctions and is considered as thirteenth dealer for the purpose of this paper.

The primary market data set contains all individual bids (price-quantity pairs) submitted by primary dealers in auctions of 52-week Treasury bills from April 2003 to September 2004 and 2-year bonds from January 2003 to December 2004. As Treasury bills are offered weekly, this time span encompasses 74 auctions. In the case of 2Y bonds sold on a monthly basis, there are 36 observations, 12 of which were supplementary, fixed price tenders. The total value of 2Y bonds sold within the analyzed period amounts to approximately PLN 62 billion (or over USD 17 billion) and this of bills, to PLN 75 billion (approximately USD 20.6 billion).

Selected statistics of the market and bidding functions of the bidders are presented in table 1.

Observations that are noteworthy from the researcher's viewpoint are the following. First, competition as measured by bid/cover ratio (the ratio of the sum of all bids to the amount sold) is not very tough. In the analyzed period the ratio was 2.43 in the case of Treasury bills and 2.63 in the case of 2Y bonds, lower than reported in most studies. It was, however, significantly higher in fixed price tenders.

Regarding the individual bids, it is clear that buyers submit non-trivial demand functions; the average number of bids submitted by the individual dealer active in a particular auction was equal to 12.6 in the case of 2Y bonds and 9.6 for 52W bills. This is a clear message that it is desirable to model individual demand functions as a downward-sloping continuous or multi-step function rather than as a single-step function. This also implies making use of the divisible goods auction theory rather than unit-demand extensions of the standard one-item auction theory.

Potential reasons for large number of bids per bidder examined in the literature include risk aversion, adjustment to winner's curse (Gordy 1999) or collusion (see e.g. Back, Zender 1993). The latter, however is restricted to the case of uniform pricing rule. Furthermore, major banks – primary dealers can be convincingly argued to display risk neutrality.³

Table 1
Selected summary statistics of the primary market data

	52W bills	2Y bonds
Number of competitive auctions	74.00	24.00
Mean bid/cover ratio	2.43	2.63
Total number of bids	8 988.00	3 356.00
Bids per auction, min.	54.00	69.00
Bids per auction, mean	118.70	145.60
Bids per auction, max.	198.00	217.00
Accepted bids per auction, min.	2.00	2.00
Accepted bids per auction, mean	60.00	62.80
Accepted bids per auction, max.	109.00	108.00
Active dealers per auction, mean	12.30	12.20
Bids per dealer in auction, mean	9.60	12.50

³ This "preference" might however not be faithfully implemented by the manager. The incentive scheme may induce risk-aversion.

Yet, the analysis of institutional features of the Polish market, supplemented by information gathered from talks with traders and representatives of the Ministry of Finance suggests another possible explanation of the large number of bids. Whereas primary dealers make purchases on their own account and the when-issued market is non-existent,⁴ every auction is preceded by meetings of the primary dealers with representatives of other large financial institutions. These convey information regarding the demand in the market and are thus very valuable from the dealers' viewpoint. On the other hand, the clients signaling early their willingness to buy the security are able to negotiate a price below the official post-auction ask price posted by the dealers.

The secondary market data was obtained from the Warsaw Stock Exchange. This is not entirely satisfactory given that it only represents a small fraction of the secondary Treasury securities market in Poland. The volume of transactions is by far the greatest on the unregulated market of negotiated inter-bank transactions. Unfortunately, no data on those can be obtained. The other segment of the secondary market, the Electronic Treasury Securities Market (ERSPW), while having higher average volumes of transactions than the WSE, is however quite often too thin, particularly for short-term securities.

The secondary market data set used in this paper contains bid and ask yields of benchmark 1-year and 2-year securities posted on the day of the auction and one (working) day before the auction. Further, to capture the level of market volatility, I compute the sample variance of logged daily price changes within 22 trading days (or approximately one month) preceding each auction.

2.1. Underpricing

Following the standard approach, I compute the mean spread between the weighted average yield of an accepted bid in the auction and the midpoint of the bid-ask spread of the benchmark security at the end of the auction day. The numbers reported in this subsection have to be treated with caution due to data problems signaled in the previous subsection. In particular, the bid-ask spread is relatively wide (around 6 basis points on average). If the average price in negotiated transactions in the inter-bank market differs systematically from the mean bid-ask spread on the WSE, the aggregate profits may be over- or under-estimated.

Underpricing measured in this way amounts to 5.41 basis points in the case of 2Y bonds and 5.7 basis points in the case of 52W bills. This translates into profit of 9.5 cents or 5.4 cents per USD 100 respectively, numbers statistically different from zero.

These figures are substantially higher than those reported in the previous studies (see e.g. Keloharju et al. 2005 for an overview). This might not be surprising given that the Polish securities market is still in its development stage. In particular, the primary dealer system was only launched in 2002. To the extent that brokers responsible for submitting demand functions are concerned about possible overbidding (which is immediately seen as a loss from the bank management's viewpoint) rather than underbidding (resulting in a less obvious loss due to missed opportunity), relative underpricing might have resulted from their willingness to deal cautiously with the new system. Investigation into possible collusion as a potential reason for substantial gap between yields in the primary and secondary markets seems difficult.

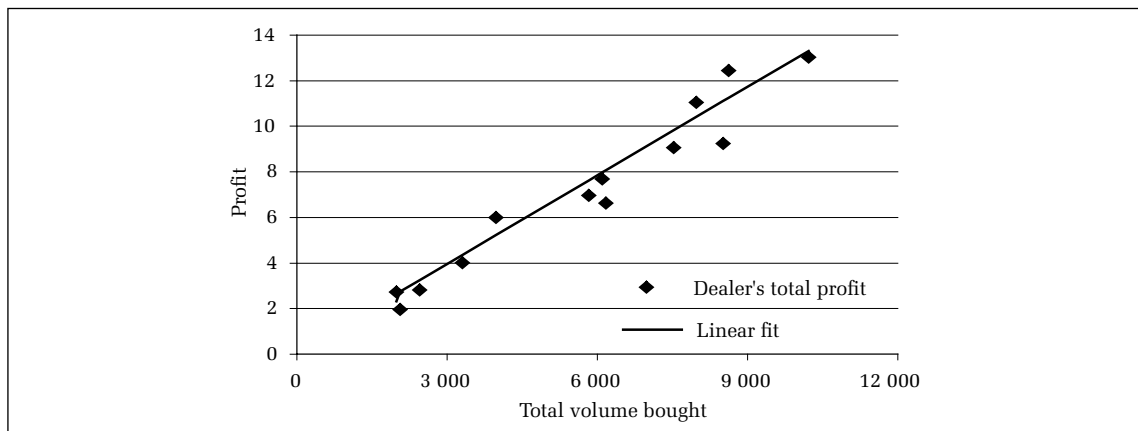
⁴ As in many other countries where relatively small market is likely to be short-squeezed.

We are also able to compute individual, dealer-specific profits, aggregating over all successful bids made by a particular dealer. Of particular interest is the relationship between profits and aggregate purchases, as it indicates to what extent smaller bidders are in a position to compete with the large ones. It is quite clear that large players have the incentive and means to pursue more detailed research regarding possible shifts in the value of the security. Further, when controlling a substantial part of the market the dealer faces relatively less uncertainty regarding the aggregate bid function. In other words, two separate entities could, in general, raise their joint profit by joining forces and thus, taking into account what previously had been an external effect. Thus, profits would be expected to grow more than proportionally with the volume of purchases.

The data does not provide substantial support for this hypothesis. Relationship between overall amount of purchases and profits in 52W bill auctions is presented in Figure 1, along with a linear approximation fitted by the ordinary least squares method. As the line fits the data rather well and can be extended to (nearly) cross the origin, it clearly suggests that dealers' rents are proportional to the amount bought. This can be confirmed by running a log-linear regression showing that elasticity of profit with respect to purchase is equal to 1.06 and not significantly different from 1 ($p = 0.339$). In the case of bond auctions, log-linear regression can only be run if we exclude a single outlier observation with negative profit,⁵ just to find that elasticity of the profit with respect to the amount purchased is 1.27, significantly more than 1 ($p = 0.032$). Without dropping any observation, we can only compute the ratio of profits to purchases and regress it on the amount purchased, concluding that the null hypothesis that dependent variable is a plain constant cannot be rejected.

To sum up, the obtained results indirectly indicate that dealers possess information of comparable precision and that they compete on roughly equal terms – additional profits owed solely to the scope of operation are not observed in bill auctions and are modest, at best, in the case of bond auctions.

Figure 1
Dealer-specific profits and volumes bought in bill auctions (in millions of PLN)



⁵ This is justified by the fact that this dealer's poor performance resulted predominantly from a catastrophic loss in a single auction early after the introduction of the system.

3. Methodology

Estimation of economic determinants of aggregated demand functions and resulting profits proceeds in following steps. First, I fit the logistic and normal cdf functions to the aggregate demand. Next, I regress the obtained parameters on economic variables that are known at the beginning of the auction. Finally, I reconstruct the expected demand functions conditional on these variables and compute the degree of underpricing and dealers' profits. Before we turn to the detailed description of the estimation procedures, let us first consider a simple model of individual behaviour that lends support to the normal cdf formulation.

3.1. Rationale for the normal cdf model

The advantage of the normal cdf specification is that it arises naturally under assumption of normality imposed on the distribution of signals available to the dealers. To illustrate the point with a simple example, we can assume that each bidder $i = 1, \dots, I$ observes a normally distributed signal y_i in the yield space⁶ with identical mean μ_0 and variance σ_0^2 . Further, each bidder submits a single bid of value A_i . I assume that this volume is independent of the signal, but possibly differently distributed for different i 's. This corresponds to the observation that in the analyzed data set the amount sought tends to be quite steady over time for each bidder, yet differs substantially among bidders. In other words, bidders seem to adjust the quantity demanded to their "capacity" and the price level to the perceived market conditions. I assume that the bid is calculated as a signal inflated by a fixed amount K . Obviously, such inflating of the yield rates corresponds to shading of prices.⁷ Then, denoting the aggregate demand at yield rate y by $D(y)$ and individual demands by $d_i(y)$, we can express the expectation of $D(y)$ in terms of probability of observing particular signals:

$$\begin{aligned} E(D(y)) &= E(\sum_{i=1}^I d_i(y)) = \sum_{i=1}^I E(A_i) Pr(y_i + K < y) = \sum_{i=1}^I E(A_i) Pr(y_i < y - K) = \\ &= E \sum_{i=1}^I A_i \Phi \left(\frac{y - K - \mu_0}{\sigma_0} \right) \end{aligned} \quad (1)$$

where Φ is the standard normal distribution function. If the number of bidders is sufficiently large, the aggregate demand function can be approximated by the scaled normal cdf specification (see also next subsection)

$$D(y) = \beta \Phi \left(\frac{y - \mu_0}{\sigma_0} \right) \quad (2)$$

where

$$\beta = \sum_i A_i$$

⁶ Throughout the paper I speak of bids, demand functions etc. being made in terms of yield to maturity rather than price. Given that the relation between the two is locally linear, this choice seems rather innocuous.

⁷ We expect that the K will be positive, reflecting the bidder's attempt to make positive profit. Shading of prices by a fixed amount is a fairly standard, if simplifying, assumption, made e.g. in Goldreich (2003). Proportional shading would lead to identical results. Note that we find, in particular, that shading is not player-specific; indeed, we do not find any hints in the data that some players consistently bid more aggressively than others.

$$\begin{aligned}\mu &= \mu_0 + K \\ \sigma &= \sigma_0\end{aligned}\tag{3}$$

In this simple environment the estimated parameters of the aggregate demand function can be readily translated into parameters of the distribution of signals observed by the players and amounts they seek to purchase.

Similar reasoning can be followed for cases of multiple bids per participant and more complex signal-contingent behaviour. In our data set, as shown previously, the assumption that bidders' information is equally precise can be sustained. Yet, the single-bid feature of the simple model sketched above, cannot. Still, if we are willing to accept the supposition that primary dealers submit multiple bids mostly based on the demand signals received from potential contractors, the following extension of the model can account for normal cdf shape of aggregate demand function.

Each bidder $i = 1, \dots, I$ observes some $n_i \in \mathcal{N}$ quantity-yield pair signals (a_{ij}, y_{ij}) , $j = 1, 2, \dots, n$ from potential investors; n_i being identically and independently distributed, y_{ij} following normal distribution with mean μ_0 and variance σ_0^2 . Regarding a_i we only assume it is positive and independent of the vector y with the typical element y_{ij} . The bidder's strategy is to submit m_i bids (suppose for simplicity that $m_i = m$ is fixed), (Y_{ik}, A_{ik}) , $k = 1, 2, \dots, m$ with yield rates being linear combinations of yield signals observed by this bidder, possibly shifted by a constant:

$$Y_{ik} = w_{ik0} + \sum_{j=1}^{n_i} w_{ikj} y_{ij}\tag{4}$$

where w_{ikj} , $j = 0, \dots, n_i$ are pre-specified weights. Further, we assume that amounts sought are functions of the observed quantity signals:

$$A_{ik} = f_i(a_{i1}, \dots, a_{in_i})\tag{5}$$

The weights w_{ikj} may be conditional on any information available prior to the auction and, obviously, depend on n_i . Functions f_i are also allowed to vary among bidders. Because submitted yield rates are linear combinations of independent normal variables, they are also normally distributed. We also note that each A_{ik} is independent of Y_{ik} . Derivation analogous to (1) leads also in this case to the conclusion that aggregate demand function can be described by a normal cdf curve.

While this formulation is quite general, it is tempting to consider a simple and intuitively appealing example. The dealer may, for instance, upon observing the signals from potential investors, compute the average signaled yield rate $\bar{y}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} y_{ij}$ and submit bids for quantities identical to those originally signaled with yield rates being weighted averages of original signals and \bar{y}_i possibly shaded by a fixed amount:

$$A_{ik} = a_{ik}\tag{6}$$

$$Y_{ik} = w_{ik0} + w_{\bar{y}_i} \bar{y}_i + (1 - w_{\bar{y}_i}) y_{ik}\tag{7}$$

for $k = 1, 2, \dots, n_i$. Amount-weighted average of the terms w_{ik0} can then be interpreted as shading. The dealer may also submit an additional bid (or bids), not aimed at any particular investor, e.g. $A_{i0} = z \sum_{j=1}^{n_i} a_{ik}, Y_{i0} = w_{k0} + \bar{y}_i$ where z denotes a positive constant. It is very easy to check that this strategy is a special case of the model given by Eq. (4) and (5).

It should be noted that this model implies that also dealer-specific individual demand curves may be approximated by normal cdf specification. Indeed, running the estimation procedure described in the following subsection for each bidder in each auction separately,⁸ I find on average a much better fit of the logistic and normal functions than of the linear-quadratic function (which also has three parameters). This analysis is, of course, problematic given a relatively small number of observations (bids per dealer). Even when this number is greater than 10, I still observe a remarkably good fit. This lends support to the simple model described above.

Of course, the normal cdf shape is obtained in the model sketched above partly due to the assumption of normality of signals. It would probably be possible to develop a similar model supporting the logistic distribution (though convolution of variables following logistic distribution is not logistic) but normality of signals (which are likely to be an aggregation of great many bits of information) is an intuitively compelling assumption.

3.2. Fitting aggregated demand functions

To compare the two approaches of approximating the aggregate demand function, namely the logistic function approach and the normal cdf approach, I first estimate the three-parameter logistic function given by the formula:

$$D(y) = \frac{\alpha}{1 + \exp\left(-\frac{y - \tau}{\lambda}\right)} + \varepsilon_L \quad (8)$$

where $D(y)$ stands for demand (aggregate bid) at given yield⁹ of the security y .

Interpretation of the parameters is as follows:

- α is the maximal demand, that is, asymptotic demand for increasing y ,
- λ is a scale parameter that captures dispersion of the bids,
- τ determines the point of inflection of the logistic curve, thus corresponds to the general yield level (and resulting price level).

To ensure that the iterated non-linear least squares estimation procedure converges to the global minimum, it is essential to start with appropriate initial values of parameters. To this end, I first estimate the linearized version of the equation given above:

$$\log\left(\frac{D(y)}{\alpha - D(y)}\right) = \frac{1}{\lambda} y - \frac{\tau}{\lambda} + \varepsilon_{LL} \quad (9)$$

⁸ Due to the scarcity of place and the fact that this analysis is of no direct practical importance, I do not reproduce the detailed results here. These are available from the author upon request.

⁹ Modeling relationship between demand and price is also customary. See footnote 1.

To transform this relationship into an estimable linear model, α has to be set a certain value. In view of the interpretation of this parameter as theoretical maximal demand, it is natural to set α equal to a number somewhat higher than the actual maximal demand (or demand for lowest price) in a given auction. While this is admittedly an arbitrary decision, I follow Preget and Waelbroeck (2005) in this respect and set α equal to exactly 1.01 of the maximal demand. It is then possible to estimate the linearized equation and obtain initial values for τ and λ :

$$\tau = -\frac{a}{b}, \lambda = \frac{1}{b} \quad (10)$$

where a denotes the estimate of intercept in Eq. (9), while b is the estimated slope.

With these initial values I perform the non-linear least squares estimation of Eq. (8) to obtain the parameters and goodness-of-fit statistics of the logistic approximation.

Next, a scaled normal cdf,

$$D(y) = \beta \Phi\left(\frac{y - \mu}{\sigma}\right) + \varepsilon_N \quad (11)$$

is fitted. Given that the interpretation of β is identical to α , I set identical initial value for this parameter. Also μ , which indicates the point of inflection, is set equal to τ obtained from the regression model (9). As for σ , one has to take into account the relationship between the variance of logistic and the standard normal distributions. If random variable X follows logistic distribution with cdf $F_x(x) = \frac{1}{1+e^x}$ then

$$VAR(X) = \frac{1}{3}\pi^2 \quad (12)$$

Thus I set initial value of σ as:

$$\sigma = \left(\frac{1}{3}\right)^{0.5} \pi \lambda \quad (13)$$

and analogously estimate parameters of the normal cdf specification by means of the iterated non-linear least squares method. The choice of this procedure stems, on one hand, from the fact that normal and logistic curves are so similar; on the other hand, there is no useful linearization of the normal model. I shall hasten to say, however, that this somewhat mechanical application of the estimation procedure, most suitable for the logistic model, to the normal model may lead to a worsened fitting of the latter, if anything.

3.3. Explaining parameters of demand functions

To be able to relate the shape of the aggregated demand functions to the underlying economic conditions and subsequently make predictions, I estimate the model by means of the Seemingly Unrelated Regression (SUR). This choice of the estimation method is justified by the fact that within-period error terms affecting the value of particular parameters of the aggregated demand functions are likely to be correlated (as these parameters are jointly determined by strategic decisions of players).

Formally, I assume the following model:

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \mathbf{u}_i, \quad E(\mathbf{u}_i \mathbf{u}_i^T) = \sigma_{ii} \mathbf{I}_n \quad (14)$$

where \mathbf{y}_i stands for n -vector of observations on the i^{th} dependent variable (parameter of the logistic or normal cumulative distribution function) and \mathbf{X}_i is a $n \times k$ matrix of explanatory variables.

As mentioned before, I allow for correlation of error terms across equations within the set time period:

$$E(u_{it} u_{jt}) = \sigma_{ii} \text{ for all } t, \quad E(u_{it} u_{sj}) = \sigma_{ij} \text{ for all } t \neq s \quad (15)$$

The Σ matrix with typical element σ_{ii} is referred to as contemporaneous covariance matrix. We further assume weak exogeneity:

$$E(U_i | X_i) = 0 \quad (16)$$

Under these assumptions, performing separate regressions for each of the explained variables yields consistent but inefficient estimates. Two important exceptions from the latter result are when the contemporaneous covariance Σ matrix is diagonal (but the diagonal elements σ_{ii} need not be identical) and when each of the X_i matrices of explanatory variables for variable y_i are identical. In both cases OLS can be shown to be numerically identical to SUR (see e.g. Davidson, MacKinnon 2004, pp. 508–509). This appears to be the case both in Preget and Waelbroeck (2005) and Özcan (2004). In the current paper, employing different sets of explanatory variables to each of the equations seems desirable given the small number of observations and, as discussed in the following section, justified on grounds of economic reasoning.

As Σ matrix is generally unknown, the SUR model must be estimated by means of Feasible Generalized Least Squares (FGLS). Alternative approach would be to make use of Maximum Likelihood estimation, based on the assumption of normality of error terms.

To assess the statistical significance of particular variables and the precision of parameters estimation in the relatively small sample at hand, I make use of the non-parametric bootstrap technique (bootstrap percentile, Efron 1981) to compute standard errors and confidence intervals (the latter based on quantiles of the bootstrap statistic distribution).

4. Results of estimation of parameters

4.1. Fitting aggregated demand functions

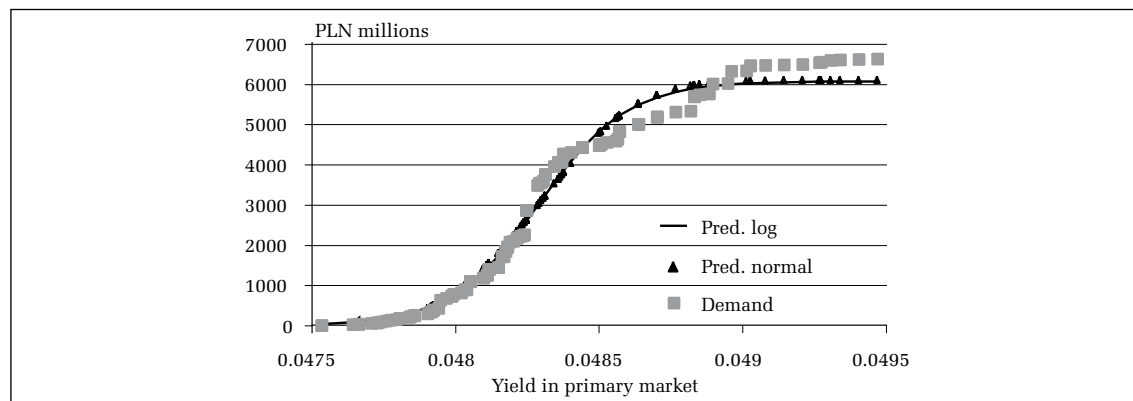
Upon running the procedure described in subsection 3.2, we note the following findings.¹⁰

First, both the logistic and normal functions perform very well in fitting the empirical demand function. The (uncentered) R^2 of the regressions is hardly ever below 0.98 in particular auctions; overall, it exceeds 0.995 in bill auctions and 0.997 in bond auctions.

¹⁰ A rather large table of auction-specific estimates and measures of goodness-of-fit, as well as diagnostic plots are available from the author upon request.

Figure 2

Actual demand and predicted values from logistic and normal CDF models, a 2Y bonds auction



Second, both models generate similar predictions. The sum of squared differences between the predicted values from the logistic model and the normal model in case of 2Y (52W bills) is close to 0.005% (0.007%) of the total sum of squares of the predicted values from the logistic model. In other words, this sum of squares amounts to about 2% of the Residual Sum of Squares (RSS) from the normal model. Consequently, we find a high correlation of over 99% between corresponding parameters of both models: α and β , τ and μ , and λ and σ .

Both of these regularities are clearly seen on a scatter plot (Figure 2) presenting the actual demand function and predictions from both models for one of the auctions. Visual inspection of the plots from the remaining auctions confirms the excellent fit.

Auction-specific differences between goodness-of-fit of the two methods are rather moderate: ratios of Residual Sum of Squares generated in the normal model to RSS from the logistic model vary from 0.72 to 1.10 in the case of 2-year bonds and from 0.84 to 1.17 for 52 week bills.

The extraordinary good fit of the normal cdf model may be further confirmed by reconstructing the cutoff-price that would emerge if the predicted demand functions are submitted under the actual volume of sales. Figures 3 and 4 show that normal cdf model makes essentially perfect

Figure 3

Q-Q plot for the actual and fitted cut-off prices in 2Y bond auctions (price in USD per USD 1 of the face value)

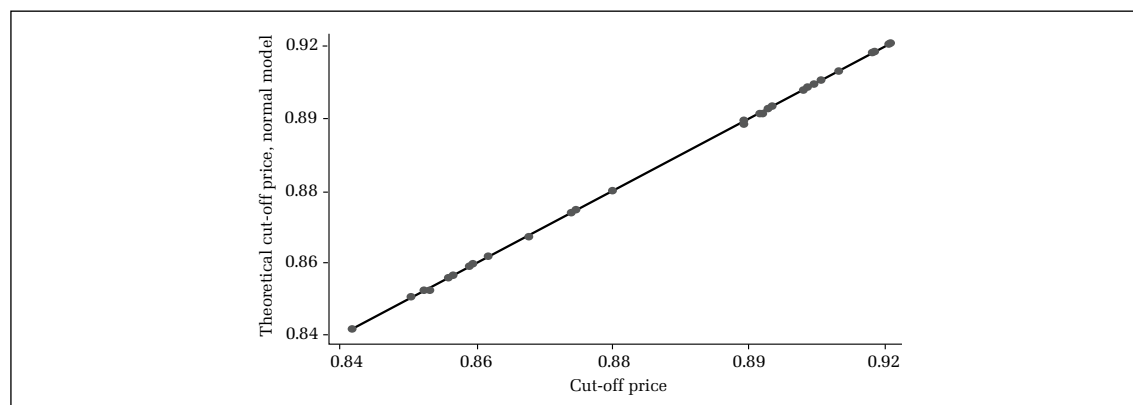
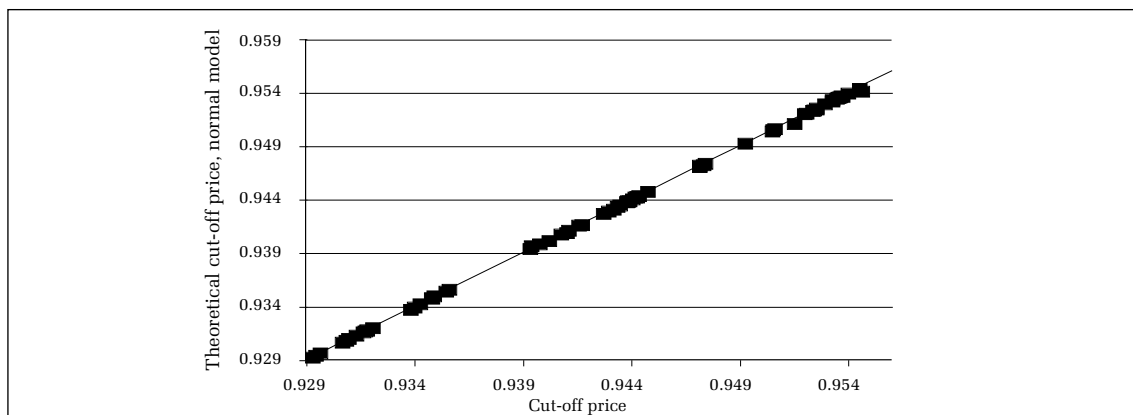


Figure 4

Q-Q plot for actual and fitted cut-off prices in 52W bills auctions (price in USD per USD 1 of the face value)



predictions of the cut-off price (here: for 2Y bonds) and the average price paid (here: for 52 weeks) respectively. Pictures for the logistic model are similar.

On average, the normal model appears to perform better in bond auctions. It generates lower residuals in 16 cases (2/3 of the sample), whereas the opposite is true in 8 auctions. The total sum of squared residuals aggregated over all auctions (which, given the common unit, seems to be a fair measure of the overall performance of the model) is some 4.7% lower in the normal model than in the logistic model. Further, as we have noted previously, the normal model appears to be “less risky” in that it generates, at worst, RSS 10% higher than the other model, whereas using the logistic model may result in inflating RSS by 39% as compared to the normal model.

In the case of Treasury bills auctions, both models perform equally well, the logistic model being fitted more closely than the normal model in exactly half (37) of the auctions. The sum of squared residuals is 1.0% lower in the latter.

We conclude that the normal cdf model, which, as shown before, has some theoretical appeal, also performs at least as well, indeed better in one of the samples, as the logistic model. Thus, it is advisable to use the normal cdf model in fitting aggregate demand functions. In the following section I focus on this approach, reporting estimation results for parameters of the normal cdf specification. Needless to say, however, all the mentioned techniques could have been equally well applied to the logistic form.

5. Economic determinants of demand function parameters

5.1. Estimation

Because one of the dependent variables¹¹ – μ – is found to be non-stationary, I subtract from it its moving average (window width equal to three observations taken with equal weights).¹² The examined set of explanatory auction-specific variables is presented in Table 2.

¹¹ I only present the results for normal model parameters. Those for the logistic model are rather similar. It should be noted that, for the sake of the estimation process, explanatory variables have been scaled in such a way so as to obtain coefficients of the same order of magnitude.

¹² I am grateful to an anonymous referee who has pointed out that omitting this step could lead to spurious results.

Table 2
Explanatory variables in SUR estimation: description, mean and standard deviation

Variable	Description	m(2Y)	sd(2Y)	m(52W)	sd(52W)
<i>a_supply</i>	volume on offer (in bill. PLN)	2.488	0.445	0.985	0.188
<i>return_wig</i>	return on WIG20 from the last auction	2.042	0.433	2.084	0.331
<i>det_a_y_1</i>	yield in sec. market, previous day ^a	0.062	0.010	0.058	0.009
<i>a_vol22</i>	volatility in sec. market ^b	1.469	3.806	1.252	4.143
<i>a_nb_01</i>	first issue of particular security	0.25	0.442	1	0

^a Detrended by subtracting moving average, constant weight, window width = 3.

^b This should not be confused with the $\frac{3}{4}$ parameter measuring the spread in the auction bids.

Table 3
Determinants of demand function parameters in 2Y bond auctions (SUR estimates)

$\beta/1000$	coeff.	st. er ^a	conf. interval ^b	
<i>a_supply</i> *10 ²	0.388	0.172	0.094	0.655
<i>a_vol</i> *10 ⁴	0.100	3.297	-0.287	0.698
<i>return_on_wig</i>	0.318	0.444	-0.015	1.26
<i>cons</i> *10	-0.414	0.414	-1.069	0.262
μ -moving average	coeff.	st. er ^a	conf. interval ^b	
<i>a_supply</i> *10 ⁶	-0.157	0.634	-1.170	0.827
<i>det_a_y_1</i> *10	0.109	0.010	0.100	0.129
<i>a_vol22</i>	-0.534	16.572	-16.645	1.188
<i>return_on_wig</i> *10 ⁻⁴	0.917	1.906	-3.618	1.813
<i>a_nb_01</i> *10 ²	-0.230	0.611	-1.267	0.697
<i>cons</i> *10 ⁻³	0.451	1.583	-1.897	3.064
σ	coeff.	st. er ^a	conf. interval ^b	
<i>a_supply</i> *10 ⁻⁶	0.024	0.158	-0.156	0.335
<i>det_a_y_1</i> *10 ⁻²	0.654	0.258	0.088	0.884
<i>a_vol22</i> *10 ⁻⁴	0.528	3.107	-0.118	1.712
<i>return_on_wig</i> *10 ⁻⁴	-0.262	0.551	-0.894	0.622
<i>a_newbond</i> *10 ⁻³	0.153	0.147	-0.094	0.384
<i>cons</i> *10 ⁻³	0.410	0.362	-0.306	0.790

^a Non-parametric bootstrap, $M = 1\ 000$ replications.

^b 90% percentile non-parametric bootstrap, $M = 1\ 000$ replications.

Table 4
Determinants of demand function parameters in 52W bill auctions (SUR estimates)

$\beta/1000$	coeff.	st. er ^a	conf. interval ^b	
$a_supply*10^{-2}$	0.109	0.045	0.035	0.184
$a_vol22*10^3$	-0.502	0.215	-0.887	-0.189
$return_on_wig*10^{-1}$	-0.123	0.198	-0.380	0.229
$cons*10$	0.143	0.047	0.065	0.219
μ -moving average	coeff.	st. er ^a	conf. interval ^b	
$a_supply*10$	0.115	0.063	0.027	0.227
$det_a_y_1*10$	0.771	0.139	0.570	1.008
a_vol22	0.356	0.173	0.092	0.648
$return_on_wig*10^{-5}$	-0.148	0.166	-0.410	0.076
$cons*10^{-2}$	-0.108	0.059	-0.216	-0.030
σ	coeff.	st. er ^a	conf. interval ^b	
$a_supply*10^{-6}$	0.361	0.171	0.070	0.624
$det_a_y_2*10^{-1}$	-0.247	0.261	-0.668	0.168
$a_vol22*10^{-1}$	-0.127	0.620	-1.004	0.949
$return_on_wig*10^{-7}$	-0.214	7.552	-1.735	13.786
$cons*10^{-3}$	0.119	0.172	-0.129	0.415

^a Non-parametric bootstrap, $M = 1\ 000$ replications.

^b 90% percentile non-parametric bootstrap, $M = 1\ 000$ replications.

The results of the estimation of the model in the sample of bond auctions and bill auctions are presented in Tables 3 and 4 respectively. As is readily seen, the total demand in bond auctions, as measured by β depends, in the first place, on the total volume offered in the auction (a_supply). This is in line with the findings reported in Preget and Waelbroeck (2005) and especially Boukai and Landsberger (1998) and Berg et al. (1999). In the latter two models, the investors bid for a fraction of the total supply rather than particular amount. The inflection point μ is best predicted by the (detrended) recent secondary market yield. Somewhat unexpectedly, this yield also contributes slightly to the variance of bids σ .

The general climate on the financial markets captured by the monthly return on WIG20 index of the Warsaw Stock Exchange has no significant effect on any parameter.

I also verify the suggestion made *inter alia* by Fleming (2002), that reopening is associated with higher borrowing cost, comparing to the first issue of a particular security. The dummy variable indicating whether the bond is issued for the first time is found to have no significant effect on the overall bids level μ .

Similar results are obtained for 52W Treasury bills. Parameter β depends heavily on the amount supplied, increasing by PLN 1.09 million for an additional PLN 1 million of the face value offered. Secondary market volatility reduces the amount sought. What regards the inflection point μ (detrended

by subtraction of its moving average), it is by far mostly determined by the (detrended) secondary market rate before the auction. The inflection point is also affected by the amount on offer and volatility of WIG20 – bidders offer lower prices when this volatility is higher. The dispersion parameter σ is positively affected by the supply offered by the Treasury – large supply of the bills offered on auction contributes to increased uncertainty regarding the optimal bidding level.

5.2. Out-of-sample prediction of the parameters of bidding functions and the resulting profits

To assess the strength of the model, I perform an out-of-sample prediction of the three parameters of aggregate demand functions and the corresponding underpricing and primary dealers' profits. To this end, I first estimate the SUR model on the sample of first 50 observations for 52W bills auctions (the sample of 2Y auctions is rather small, rendering predictive power tests almost infeasible). Basing on the actual values of the explanatory economic variables and estimated coefficients, I compute the predicted values of β , μ and λ . Table 5 reports the results.

We conclude that the model generates, on average, roughly correct predictions for all of the three parameters of the bidding functions. Table 5 also displays validity of the predictions at a given level, i.e. the fraction of the predicted values that fell into a certain interval around the actual value. Prediction of the inflection point μ is very good, 96% of the out-of-sample predictions being between 95% and 105% of the real value. Forecasts of other parameters are somewhat less precise, particularly, we note the poorly predicted standard deviation of the scale parameter β , which is largely due to two outlier observations (one with very high demand, the other with very low one). On the whole, however, the model can be said to deliver unbiased and reasonably accurate forecasts of key characteristics of the bidding behaviour and auction results.

Table 5
Out-of-sample prediction of dealers' profits and demand function parameters

$\beta/1000$	Observed	Predicted
mean	2.276	2.680
std. dev	0.725	0.182
validity at 20%	33%	x
μ-moving average	Observed	Predicted
mean	0.071	0.071
std. dev.	0.004	0.005
validity at 5%	96%	x
σ	Observed	Predicted
mean	0.375	0.522
std. dev.	0.136	0.088
validity at 20%	25%	x

6. Conclusions

This paper analyzes the unique data set from the Polish primary and secondary Treasury securities markets. Some interesting features of the data such as large numbers of bids per bidder, low cover/bid ratio and high underpricing are found. I offer some interpretation of these findings in terms of institutional characteristics of this emerging market. The paper also introduces a modified approach to modeling aggregate demand functions in Treasury auctions, based on the normal cdf rather than the standard logistic formulation. In the analyzed data, the former appears to slightly outperform the latter in terms of goodness of fit. The reasonably accurate prediction (despite a rather small sample at hand) of the parameters of the normal cdf specification based on economic variables known before the auction makes forecasts of the auction results possible. This enables the economists (and the Treasury alike) to monitor the performance of the auction design used. Any substantial and systematic deviation from the predicted shape of the demand functions and the corresponding profit obtained by the primary dealers should induce an in-depth investigation and consideration of possible institutional changes. Particularly in the case of collusive agreement, an early detection of the resulting underpricing is essential to avoid huge losses by the Treasury. Finally, the model delivers a powerful tool for an analysis of the impact of any possible change in the design of auctions on dealers' behaviour and the corresponding cost of public debt servicing.

References

- Back K., Zender J. (1993), Auctions of Divisible Goods: on the Rationale for the Treasury Experiment, *Review of Financial Studies*, 6(4), 733–764.
- Berg S.A., Boukai B., Landsberger M. (1999), *Bid Functions for Treasury Securities; Across Countries Comparison*, mimeo.
- Boukai B., Landsberger M. (1998), *Market bid functions for Treasury securities as logistic growth curves*, Working Paper, Indiana University – Purdue University, Indianapolis, Indiana.
- Cammack E.B. (1991), Evidence of bidding strategies and the information in Treasury bill auction, *Journal of Political Economy*, 99 (11), 100–130.
- Daripa A. (2001), A theory of Treasury auctions, *Journal of International Money and Finance*, 20, 743–767.
- Davidson R., MacKinnon J. (2004), *Econometric Theory and Methods*, Oxford University Press, New York.
- Efron B. (1981), Censored Data and the Bootstrap. *Journal of the American Statistical Association*, 76 (374), 312–319.
- Fevrier P., Preget R., Visser M. (2002), *Econometrics of Share Auctions*, CREST Working Paper, 2002-09.
- Fleming M.J. (2002), Are larger Treasury issues more liquid? Evidence from bill reopenings, *Journal of Money, Credit and Banking*, 34 (3), 707–735.
- Goldreich D. (2003), Underpricing in discriminatory and uniform-price Treasury auctions, Working Paper, December, London Business School.

- Gordy M.B. (1999), Hedging Winner's Curse with Multiple Bids: Evidence From The Portuguese Treasury Bill Auction, *Review of Economics and Statistics*, 81(3), 448–465.
- Hamao Y., Jegadeesh N. (1998), An Analysis of Bidding in the Japanese Government Bond Auctions, *Journal of Finance*, 53, 755–772.
- Heller D., Lengwiler Y. (2001), Should the Treasury price discriminate or not? A procedure for computing hypothetical bid functions, *Journal of Institutional and Theoretical Economics*, 157(3), 413–429.
- Hortaçsu A. (2002a), *Bidding Behavior in Divisible Good Auctions: Theory and Evidence from the Turkish Treasury Auction Market*, Working Paper, Stanford University.
- Hortaçsu A. (2002b), *Mechanism choice and strategic bidding in divisible good auctions: an empirical analysis of the Turkish Treasury auction market*, Working Paper, University of Chicago.
- Jegadeesh N. (1993), Treasury auction bids and the Salomon squeeze, *Journal of Finance*, 48, 1403–1419.
- Keloharju M., Nyborg K.G., Rydqvist K. (2005), Strategic Behavior and Underpricing in Uniform Price Auctions: Evidence from Finnish Treasury Auctions, *Journal of Finance*, 60, 1865–1902.
- Nyborg K.G., Sundaresan S. (1996), Discriminatory versus uniform Treasury auctions: Evidence from when-issued transactions, *Journal of Financial Economics*, 42, 63–104.
- Nyborg K.G., Rydqvist K., Sundaresan M. (2002), Bidder Behavior in Multi-Unit Auctions. Evidence from Swedish Treasury Auctions, *Journal of Political Economy*, 110, 394–424.
- Özcan R. (2004), *The Logistic Function Approach to Discriminatory and Uniform Price Treasury Auctions*, <http://idei.fr/doc/conf/ea/ozcan.pdf>.
- Preget R., Waelbroeck P. (2005), Treasury Bill Auction Procedures: Empirical Perspectives from French Market Bid Functions, *Journal of International Money and Finance*, 24 (7), 1054–1072.
- Simon D.P. (1994), The Treasury's experiment with single-price auctions in the mid-1970s: winner's or taxpayer's curse?, *Review of Economics and Statistics*, 76, 754–760.
- Umlauf S.R. (1993), An empirical study of the Mexican Treasury bill auction, *Journal of Financial Economics*, 33, 313–340.
- Vargas J.S. (2003), *Bidder Behavior in Uniform Price Auctions. Evidence form Argentina*, mimeo.
- Wang J.J.D., Zender J.F. (2002), Auctioning Divisible Goods, *Economic Theory*, 19, 673–705.
- Wilson R. (1979), Auctions of Shares, *Quarterly Journal of Economics*, 93 (4), 675–689.

